Estimating Annual Growth Response to a Forest Treatment

Om å estimere årlig endring i grunnflatetilvekst ved en skogbehandling

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Abstract.

MØNNESS, E 1997. Estimating Annual Growth Response to a Forest Treatment. (*Om å estimere årlig endring i grunnflatetilvekst ved en skogbehandling.*) Medd. Skogforsk. 48(14): 237-254.

Several estimators of response to a forest treatment are compared. All estimators are evaluated theoretically on a common basis. Even though they are based on different models, they appear to have a similar structure. Based on uniformity trials, they are also compared empirically by means of the mean square error. The unadjusted response estimator, that is the annual basal area growth (increment) difference between the treated and the control plots, has an annual root of mean square error of approximately *0.1 m^2 /hectare*. The other estimators use preperiod increment difference to improve the response estimate. The easily calculated difference and ratio estimators are satisfactory, with root of mean square *errors ≈0.04 m^2 /hectare* initially, rising to about *0.1 m^2 /hectare* about *20* years after the preperiod. Estimators using covariance, time series or regression adjustment do not improve upon this. The data has its origin from a broad set of fertilization field trials on *Pinus Sylvestris L. and Picea Abies L. Karst.* in the south east of Norway.

Key words: Long term prediction error, uniformity trials.

Utdrag.

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Flere estimatorer av respons til en skogbehandling er sammenliknet. Alle estimatorene er sammenliknet på en sams teoretisk måte. Til tross for ulikt utgangspunkt viser de seg å ha en lik struktur. Estimatorene er også sammenliknet empirisk ved hjelp av blindforsøk. Den ujusterte estimatoren, differansen i årlig grunnflatetilvekst mellom en behandlet rute og en kontrollrute, har en årlig middelfeil på ca. *0.1 m^2/hektar*. De øvrige estimatorene korrigerer estimatet basert på tilveksten i forperioden. De enkle metodene basert på konstant differanse eller konstant forhold, har en middelfeil på ca. *0.04 m^2/hektar* de første forsøksårene, økende til ca. *0.1 m^2/hektar* etter ca. 20 år. Estimatorer som gjør bruk av kovariansanalyse, tidsrekkeanalyse eller regresjon reduserer ikke middelfeilen ytterligere.

Dataene har sitt utspring i gjødslingsforsøk på *Pinus sylvestris L.* og *Picea abies L. Karst* i Sydøst Norge.

Nøkkelord: Langsiktig prediksjonsfeil, blindforsøk.

Preface.

I am very proud of being allowed to try to honour   J. Eid, S. Nersten and A. Svendsrud with this article. It is a rewritten version of MØNNESS, (1991), incorporating ideas I developed as a referee of McWILLIAMS & BURK (1993).
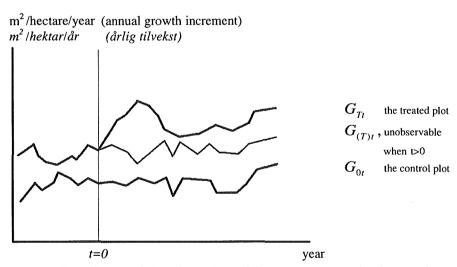
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Introduction

The theory has been developed in connection with fertilization field experiments. However, fertilization is not at all of vital importance to the theory. Any situation where growth is to be measured, and where some treatments are to be compared on some plots or individual trees, may profit from ideas presented here. The presentation is therefore written in a general manner.
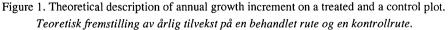
How to Measure a Treatment Response. Why is it a Problem?

A typical experiment is about comparing a new treatment, a new regime, with a standard version, or with no treatment at all (a control). There are a huge amount of standard experimental designs available to the experimenter, say analysis of variance (ANOVA), analysis of covariance (ANCOVA), regression. Why is this not enough?

In forestry, field experiments are typically long-lasting. A certain treatment may have an effect on only a stage in a tree's development. The experimenter is often forced to accept an experimental playground involving trees already grown for years. The effect on the final product may be indirect or out of reach of the experimenter. This is of great contrast to agriculture, where the entire life span of the experimental objects is one season only.

The trees will of course grow even if not treated. What is usually of interest is the *additional* growth/yield due to the treatment, often called the *response. The response to the treatment* is the difference between what is gained and what would have been gained if not treated.

m^2/hectare/year (annual growth increment)
m^2 /hektar/år   (årlig tilvekst)

G_{Tt} the treated plot

$G_{(T)t}$, unobservable
           when t>0

G_{0t} the control plot

t=0   year

Figure 1. Theoretical description of annual growth increment on a treated and a control plot.
  Teoretisk fremstilling av årlig tilvekst på en behandlet rute og en kontrollrute.

Notation

Indexes

it	i identifies a certain field plot, t is the time (year). In an actual setting, i may be a multiindex, dependent of experimental design.
$i = 0$	indicates no treatment; a control plot.
$i = T$	indicates treatment.
$i =(T)$	indicate a treated plot, but the value that would have been if not treated. (This value is usually unobservable)
t	can be both negative and positive.
$t=0$	is the time of experimental establishment, when the treatment was first applied.
$t=negative$	is observations, or records of growth, before experimental establishment. This period is called the *preperiod*. If $t= negative$, then values associated with $i = T$ and $i =(T)$ are identical.
$t=1$	is first year a treatment response could occur.

Main Symbols

G_{it} observed basal area growth (increment) on plot i in year t (m^2/hectare/year). If t is explicit equal to 5, G_{i5} is the total growth in the entire 5 year period.

Y_{it} observed basal area (yield) on plot i in year t (m^2/hectare).

T_{it} is the effect, or the response of the treatment, either on yield or growth. The growth response is $T_{Tt} = G_{Tt} - G_{(T)t}$

$F_t = G_{(T)t} - G_{0t}$ Thus F_t is the difference in growth between two plots in year t, if no treatment had been applied in any of the two plots. F_t may be seen as the random variation in growth between any two plots. We suppose these two plots are comparable, they are supposed to be in some common block or in some common field. The effect of varying climate is small on F_t since it is a difference.

Mean values have a bar above the symbol. Typically,

$$\overline{G}_{it} = \frac{1}{w} \sum_{v=t-w+1}^{t} G_{iv} \text{ , with } w \text{ equal to } 5 \text{ or } 10.$$

Estimated or predicted values have a ^ above the symbol.

e	random error
$E(\cdot)$	expectation.

MSE mean square error. *MSE* includes both variance and bias. On uniformity trials the true value to be evaluated is 0, thus the *MSE* is calculated as

$$\frac{1}{k} \sum_{v=1}^{k} X_v^2$$ where X_v is the statistic in question. k is the number of observations available to the calculations.

$RMSE$ square root of *MSE*

$U_{Tt}, D_{Tt}, R_{Tt}, COV_{Tt}, REG_{Tt}$ and AR_{Tt} are different estimators of treatment response introduced in the next chapter.

Treatment Response Estimators
Introduction

Consider an ordinary analysis of variance (ANOVA) of yield at year $t=5$:

$Y_{i5} = \mu + \mu_{i5} + e_{i5}$. ($i$ can have more structure, depending on the experimental design.) The response to the treatment is estimated as the contrast $Y_{T5} - Y_{05}$: The simple difference between a treated plot and a control plot, (or means if there are replicates).

Consider instead analyzing the growth with an ANOVA scheme. $G_{i5} = \mu + \mu_{i5} + e_{i5}$. The treatment contrast is as above $G_{T5} - G_{05}$. But the yield at $t=5$ is the sum of the yield at $t=0$ plus the growth in time 0-5. Let T_{T5} be the additional yield which is also equal to the additional growth. We then have

$G_{T5} - G_{05} = T_{T5} + (G_{(T)5} - G_{05})$, and

$Y_{T5} - Y_{05} = T_{T5} + (Y_{(T)5} - Y_{05}) = T_{T5} + (G_{(T)5} - G_{05}) + (Y_{T0} - Y_{00})$.

Thus: Analyzing the growth with an ordinary ANOVA gives an estimate of the treatment effect biased with the difference in growth between the treated and untreated plot that will occur in the trial period independent of any treatment. Analyzing the yield will bias the estimate with still another term; the difference in yield at the establishment of the experiment. I consider this as a strong argument not to analyze yield, but the growth. The dependence of the yield difference by pretreatment yield difference was also experienced by LIPAS (1979), and confirmed by MCWILLIAMS & BURK (1994). Their simulation work shows a strong linear correlation between bias and pretreatment yield, and with the same actual size as documented theoretically here.

The Unadjusted Estimator

We call the growth difference introduced above the *unadjusted* estimator:

$U_{Tt} = G_{Tt} - G_{0t} = T_{Tt} + (G_{(T)t} - G_{0t}) = T_{Tt} + F_t$

Thus bias and MSE of U_{Tt} are associated with F_{Tt} :

$E(U_{Tt} - T_{Tt})^2 = E(F_t)^2$.

In order to be a good estimator, $G_{(T)t}$ and G_{0t} on figure 1 should nearly coincide.

The Difference Estimator [1]

Assume the difference in increment between two plots in the same block to be nonzero, but constant in time t, at least for the preperiod and the experimental period. This means that the curves $G_{(T)t}$ and G_{0t} on figure 1 should be nearly

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[1] In my earlier work I called this estimator «the constant estimator».

equidistant. The preperiod then gives a prediction of F_t, leading to the following estimate of response:

$$D_{Tt} = G_{Tt} - G_{0t} - (\overline{G}_{T0} - \overline{G}_{00}) = T_{Tt} + F_t - \overline{F}_0$$

and

$$E(D_{Tt} - T_{Tt})^2 = E(F_t - \overline{F}_0)^2$$

$\overline{F}_0$ should be a good predictor of F_t. This is reasonable for short time periods but perhaps not for long time periods.

The Ratio Estimator

Define

$$R_{Tt} = G_{Tt} - G_{0t} \cdot \frac{\overline{G}_{T0}}{\overline{G}_{00}} = T_{Tt} + G_{(T)t} - G_{0t} \cdot \frac{\overline{G}_{T0}}{\overline{G}_{00}}$$

and after some algebra

$$R_{Tt} = T_{Tt} + F_t - \overline{F}_0 \cdot \frac{G_{0t}}{\overline{G}_{00}}.$$

We also have

$$E(R_{Tt} - T_{Tt})^2 = E(F_t - \overline{F}_0 \cdot \frac{G_{0t}}{\overline{G}_{00}})^2$$

The idea is that changes in annual increment between plots have a constant ratio rather than a constant difference. This means that the curves $G_{(T)t}$ and G_{0t} on figure 1 should have a proportional course. This estimator have been in use in several Swedish works (CARBONNIER (1962), ROSVALL (1979)). Both LIPAS (1979) and MCWILLIAMS & BURK (1993), report that the ratio estimator overcompensates for the initial differences.

The Covariance Estimator

A possible improvement of D_{Tt} is to annually estimate the influence of the adjustment:

Let

$$COV_{Tt} = G_{Tt} - G_{0t} - \hat{\beta}_t \cdot (\overline{G}_{T0} - \overline{G}_{00}) = T_{Tt} + F_t - \hat{\beta}_t \cdot \overline{F}_0$$

and

$$E(COV_{Tt} - T_{Tt})^2 = E(F_t - \hat{\beta}_t \cdot \overline{F}_0)^2$$

This model could give a smooth path from D_{Tt} (with $\widehat{\beta}_t = 1$) to U_{Tt} (with $\bar{\beta}_t = 0$), depending on t. This would allow for correction when there is a correlation and no correction otherwise.

COV_{Tt} is based upon the covariance model (for each t and each site):

$G_{it} = \mu_t + \mu_{it} + \beta_t \cdot \overline{G}_{i0} + e_{it}$. [2]

COV_{Tt} is the ordinary treatment contrast.

The AR Estimator

Assume that $F_t = G_{(T)t} - G_{0t}$ is an autoregressive time series of first order, $AR(1)$, with an non-zero mean (Box & Jenkins, 1976). Thus $F_t - \xi = \beta \cdot (F_{t-1} - \xi) + e_t$. The parameters ξ and β must be estimated in the preperiod, and the series is then predicted ($\widehat{F}_t$) for $t>0$.

We have

$$AR_{Tt} = G_{Tt} - G_{0t} - \widehat{F}_t = T_t + F_t - \left(\bar{\beta}^{\ t} \cdot (F_0 - \overline{F}_0) + \overline{F}_0\right)$$

and after some calculations

$$E(AR_{Tt} - T_t)^2 = E\left(F_t - \overline{F}_0 - \bar{\beta}^{\ t} \cdot (F_0 - \overline{F}_0)\right)^2$$

(F_0 is the growth difference in year 0, $\overline{F}_0$ is the mean growth difference in the preperiod.) AR_{Tt} will converge to D_{Tt} as t in-creases[3], and could only be expected to be superior when t is less than, say, 5 years. In a separate unpublished investigation, I have selected the $AR(1)$ model among $AR()$ models with lags ranging from one to five years.

The Regression Estimator

Consider the following ordinary regression between the increment on a treated plot and a control plot:

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[2] Instead of picking up β_t from each individual experiment, β_t could be calculated based on several experiments. Also, MCWILLIAMS & BURK (1994), consider a covariance estimator based on yield, called $R5$. But

$R5_{T5} = Y_{T5} - Y_{05} - \beta (Y_{T0} - Y_{00}) = G_{T5} - G_{05} + (1 - \beta)(Y_{T0} - Y_{00})$. Thus, this is the same as an ANCOVA model on growth using yield at establishment as a covariate instead of using growth. A model with yield will appear to have a very high correlation- squared since Y_{i0} is a common term both in the dependent and independent variable.

[3] Since $0 < \beta < 1$, $\bar{\beta}^{\ t}$ will converge to zero as t increases. Any $AR()$ model will converge to its mean when predicting over several time periods.

$$G_{(T)t} = \mu + \beta \cdot G_{0t} + e_t$$

On the preperiod this regression is observable and the parameters are estimated independently for each pair of treated/control plot.

From theory we know that the least square estimates from any regression are functionally dependent by $\hat{\mu} \equiv \overline{G}_{T0} - \hat{\beta} \cdot \overline{G}_{00}$

Then

$$REG_{Tt} = G_{Tt} - \hat{G}_{(T)t} = G_{Tt} - \hat{\mu} - \hat{\beta} \cdot G_{0t} = T_t + G_{(T)t} - (\overline{G}_{T0} - \hat{\beta} \cdot \overline{G}_{00}) - \hat{\beta} \cdot G_{0t}$$

and after some calculations

$$REG_{Tt} = T_t + F_t - \overline{F}_0 + (1 - \hat{\beta}) \cdot (G_{0t} - \overline{G}_{00})$$

giving

$$E(REG_{Tt} - T_t)^2 = E\Big(F_t - \overline{F}_0 + (1 - \hat{\beta}) \cdot (G_{0t} - \overline{G}_{00})\Big)^2$$

The regression estimator is the logical estimator combining ideas from the difference and the ratio estimators. If $\hat{\beta} = 1$ then $REG_{Tt} \equiv D_{Tt}$ and if $\hat{\mu} = 0$ it can be shown that $REG_{Tt} \equiv R_{Tt}$.

In hindsight we see that all the proposed growth estimators have the same structure:

$\hat{T}_{Tt} = G_{Tt} - \hat{G}_{(T)t}$: Try to predict the growth of the treated plot if not treated ($\hat{G}_{(T)t}$), then subtract this value from the observed growth on the treated plot (G_{Tt}). U_{Tt} was the ANOVA estimator. All other estimators try to compensate for different initial growth between the plots. COV_{Tt} is the ANCOVA estimator with pretreatment growth as a covariate. D_{Tt} and R_{Tt} try to compensate in a straight forward and easy manner. All these estimators need only *one* preperiod growth observation (say, mean of the last *5* or *10* years). REG_{Tt} and AR_{Tt} operate independently for each treated plot but dependent on time *t*. In order to estimate parameters they need annual growth observations in the preperiod. I would recommend at least *10* years, but *5* years may be acceptable. D_{Tt} is a special case of COV_{Tt}, REG_{Tt} and AR_{Tt}. R_{Tt} is a special case of REG_{Tt}.

There are several other ways regression can be applied. Both LIPAS [4](1979) and MCWILLIAMS & BURK, (1994), explore this.

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[4] Lipas' regression is actually an analysis of covariance.

Estimators: Summary

All error terms are formulated in the same manner, with

$F_t - \overline{F_0} = (G_{(T)t} - G_{0t}) - (\overline{G}_{T0} - \overline{G}_{00})$ as the «standard correction».

Estimator	Error term	Comment
U_{Tt}	$F_t - \overline{F_0} \quad + \overline{F_0}$	Actually no correction.
D_{Tt}	$F_t - \overline{F_0}$	«Standard correction»
R_{Tt}	$F_t - \overline{F_0} \quad + \left(1 - \dfrac{G_{0t}}{\overline{G}_{00}}\right) \cdot \overline{F_0}$	If the growth in the control plot has a small variation, we have $R_{Tt} \approx D_{Tt}$
COV_{Tt}	$F_t - \overline{F_0} \quad + (1 - \hat{\beta}_t) \cdot \overline{F_0}$	Same structure as in R_{Tt}
REG_{Tt}	$F_t - \overline{F_0} \quad + (1 - \hat{\beta}) \cdot (G_{0t} - \overline{G}_{00})$	β depends only on the preperiod.
AR_{Tt}	$F_t - \overline{F_0} \quad - \hat{\beta}' \cdot (F_0 - \overline{F_0})$	The entire extra correction depends only on the preperiod.

The three β s have different formulas that may be found in a standard textbook .

As indicated above, the estimators $U_{Tt}, D_{Tt}, R_{Tt}, COV_{Tt}, REG_{Tt}$ and AR_{Tt} are all of the structure $\hat{T}_{Tt} = G_{Tt} - \hat{G}_{(T)t}$. More specific, their difference is how $\hat{G}_{(T)t}$ is calculated. $\hat{G}_{(T)t}$ is based on the control plot, and on the treated plot in the preperiod[5]. Also, the observation involving the treatment, G_{Tt}, enters into the estimators in the same manner. Thus the difference between the estimators is independent of the data that might have been treated, and the difference is independent of the actual size of the treatment response . But in order to calculate the *MSE* of one estimator we must know the actual value, which is usually unknown. However, the level might be zero.

Empirical Evaluation: Uniformity Trials

A *uniformity trial* consists of two neighboring plots, one of which is designated the control plot and the other the treated plot. However, no treatment is actually applied, so the true response to the hypothetical treatment is zero. Zero is the only value of a response that actually can be thought of as known. The efficiency (in terms of mean square error) of each estimator can be compared at this zero level response.

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[5] COV_{Tt} is an exception. The estimate of the covariance parameter β_t also involves G_{Tt}, and may be dependent of treated observations if the covariate is not orthogonal on the experimental design.

However, the ranking of the estimators must be valid also in real treated experiments.

The method of uniformity trials is a technique which explores the influence of natural variation on estimators rather than the actual estimates themselves. Uniformity trials were often used early in this century to study the validity of agricultural field trials. TIPPETT (1931), and FISHER (1925) use empirical results from uniformity trials in their argumentation for randomization and experimental design. AASTVEIT (1982), gives several references to uniformity trials in agricultural experiments.

Other ways can be used to compare methods: A theoretical comparison can be done by introducing a common statistical growth model, and then actually calculate the $E(\bullet)$s. This is straightforward but tedious. However, the results can still be hard to compare. Even worse is that a statement like «A is better than B» does not tell «how much better» or «how big is the actual error to worry about»?

A comparison can be done with simulation. This has been done by MCWILLIAMS & BURK (1994). The problem with simulation is that all trees have to be «grown» with some functions. One can never tell to which degree the results reflect characteristics of the functions themselves. LIPAS (1979), presents comparisons of methods based on actually fertilized plots. If estimation methods are to be compared on actually treated plots, there is a need to both estimate the treatment effect and to compare methods. Estimating treatment effect can result in underestimation of the bias; the bias will become a part of the treatment estimate. A method having a high *MSE*, due to bias, may be seen with a low standard deviation and therefore misjudged to be a good method.

Data

The data for this study were selected from Norwegian experiments with N -, NPK - fertilization on old stands of Pinus Sylvestris, (L) and Picea Abies, (L) Karst, mainly located in the southeast of Norway. There are about 70 sites each with randomized block design experiments. Even though experiments are blocked, each site is selected to minimize within site variation. Each experiment typically consists of 4 blocks with a certain number of plots (e.g. treatments) in each block, giving typically 4 replicates.
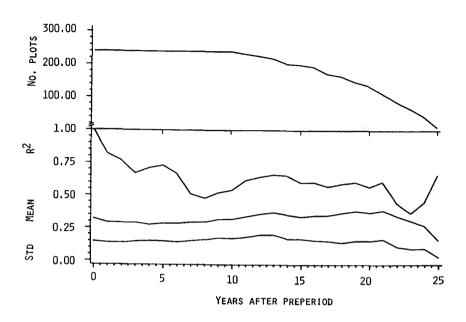
There is at least one untreated control plot in each block. The plots were mainly *0.02* or *0.04* hectare excluding a buffer zone. Cores were taken on each tree to measure annual basal area increment. The increments cover *10* years before first fertilization and up to *20* years after. A portion of these data has been reported in BRANTSEG ET. AL (1970). A data base has been established consisting of about *42000* trees on *2200* plots. General information describing the control plots is given in Table 1.
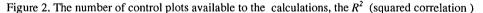
Table 1. Mean values (per hectare) and quantiles of the control plot data.
   Middelverdier per hektar og kvantiler av data fra kontrollrutene.

Variable	# obs	Mean	Quantiles				
			Min	25%	median	75%	Max
Year of establishment	243	1962	1958	1960	1961	1963	1971
Duration of experiment	243	14.6 years	6.0	12	16.0	18.0	20.0
Height above sea level	243	241m	10	130	180	320	750
Longitude	243	61.9°	59.0	60.3	60.8	64.0	66.4
Latitude	243	11.5°	7.8	10.9	11.6	12.0	14.8
Site Index (top height at age 40)	243	11.3 m	2.8	9.0	11.0	14.0	21.0
Age at breast height, $t=0$	243	87 years	19	70	88	109	220
Basal area per hectare, $t=0$	243	19.3 m^2	5.3	14.3	18.6	23.9	43.9
Volume per hectare, $t=0$	243	157 m^3	33	101	148	199	528
Number of trees/ha	243	669	150	450	625	850	1650
$\overline{G}_{i0}$ (per hectare)	243	0.29 m^2	0.05	0.19	0.25	0.34	0.76
$\overline{G}_{i5}$ (per hectare)	243	0.30 m^2	0.06	0.17	0.28	0.39	0.89
$\overline{G}_{i10}$ (per hectare)	243	0.34 m^2	0.06	0.22	0.29	0.45	0.81

As the data were not designed to fit uniformity trials, a careful study to select appropriate subsets was done. Assuming a preperiod of 5 years, the development of the control plots in the following 25 years is shown in figure 2. The annual variation of increment in a single trial is larger than the one appearing in figure 2, because figure 2 has years after treatment as time scale and not actual year. The following data sets were extracted:

Figure 2. The number of control plots available to the calculations, the R^2 (squared correlation)
between preperiod growth (mean of 5 years) $\overline{G}_{00}$ and annual increment G_{0t} , mean
annual increment (m^2 /hectare) and their annual standard deviation. Unit on lower vertical
axis is m^2/hectare/year.

*Antall kontrollruter som er tilgjengelig for beregningene, R^2 mellom gjennomsnittlig
tilvekst i forperioden og årlig tilvekst, gjennomsnittlig årlig tilvekst og deres standard
avvik. Akseenhet på nedre vertikale akse er m^2/hektar/år.*

Data Set One

Every plot has a preperiod of *10* years. If a hypothetical "fertilization" is imposed at
year *5*, then each pair of plots is a uniformity trial with a 5-year trial and a *5* year
preperiod. In each block one plot is considered to be the control; the others are
considered "fertilized". This makes 1817 uniformity trials of 214 blocks on 69 sites.
The mean responses after 5 years on each plot were analyzed. The responses within a
block are correlated due to their common control plot. This data set maintains the
pattern of the original fertilization experiments. Thus, both within-block and
between-block variation can be explored.

Data Set Two

In *28* blocks there are *2* control plots. If we consider one of them to be "fertilized"
after a preperiod of *10* years this gives us *28* independent uniformity trials.
Responses were estimated annually for up to *20* years. Data set *2* is the most obvious
set of uniformity trials contained in the material.

Data Set Three

Due to the experimental layout, the between-block variation may not be of any significance. Thus the variation between adjacent blocks may be neglected when compared to within-block variation. (This assumption will be seen to be acceptable based on results from data set no. 1). Thus the control plot from one block can serve as "fertilized" while the control plot of the adjacent block remains a control plot. This set consists of *107* independent uniformity trials. The preperiod is taken to be *5* years and the responses were estimated annually for up to *25* years. The reason that this data set is selected is the high number of independent trials and the time span available.

In each trial one plot must be decided to serve as control and the other as fertilized. This has been done in an automatic, noninspected manner based on plot number. Thus the original randomization of the experiments is in action.

Results

The increment $\overline{G}_{T5}$, and the responses $\overline{U}_{T5}$, $\overline{D}_{T5}$ and $\overline{R}_{T5}$ were calculated using the data set no. 1. Variance components for the model *Variable=site+block(site) +plot error* are given in Table 2. The main variation in the increment $\overline{G}_{T5}$ is due to site variation which is of no interest here. The site variance of the unadjusted estimator $\overline{U}_{T5}$ is totally removed. Instead there is a new block variance of the same size as the plot variance. This is due to the common control plot in each block. The estimator $\overline{D}_{T5}$ and $\overline{R}_{T5}$ also have a block variance but the total variance is severely reduced (*RMSE ≈0.04*).

Table 2. Variance component analysis.
  Varians komponent analyse.

Source	d.f.	$\overline{G}_{T5}$	$\overline{U}_{T5}$	$\overline{D}_{T5}$	$\overline{R}_{T5}$
Total σ, RMSE		0.127	0.091	0.045	0.041
Total σ^2	1816	0.016	0.008	0.002	0.002
Site σ^2	68	0.011	0.000	0.000	0.000
Block(site) σ^2	145	0.001	0.004	0.001	0.001
Error σ^2	1603	0.004	0.004	0.001	0.001

To find $\overline{COV_{T5}}$ the $\overline{G}_{i5}$ s are analyzed by the model

$\overline{G}_{i5} = \mu + \mu_{site} + \beta \cdot \overline{G}_{i0} + \beta_{site} \cdot \overline{G}_{i0} + e$.

Calculating the covariance responses

$\overline{COV_{T5}} = \overline{G}_{T5} - \overline{G}_{05} - \hat{\beta}_{site} \cdot (\overline{G}_{T0} - \overline{G}_{00})$ gives *RMSE = 0.041.*

Adjusting the $\overline{G}_{T5}$ with a common covariance (β equal on all sites) also gives *RMSE=0.041* on the response $\overline{G}_{T5} - \overline{G}_{05} - \beta \cdot (\overline{G}_{T0} - \overline{G}_{00})$. Thus $\overline{COV}_{T5}$ has the same *RMSE* as $\overline{D}_{T5}$ and $\overline{R}_{T5}$ (from table 2).

It appears that the assumption made to establish the data set no. 3 of uniformity trials is justified since the *Block(site)* variance of $\overline{G}_{T5}$ is small.

Estimated annual *RMSE*s of U_{Tt}, D_{Tt}, R_{Tt}, REG_{Tt} and AR_{Tt} based upon data set no. 2 are given in Figure 3. The unadjusted estimator U_{Tt} has stable *RMSE*s independent of t which is to be expected. All the other methods have a common development: The *RMSE*s are low at $t=1$, but increase with t and reach $RMSE(U_{Tt})$ when t is about *13*. On data set no 3 the preperiod is *5* years, giving *25* years of prediction. The methods U_{Tt}, D_{Tt}, COV_{Tt} and R_{Tt} are compared (Figure 4). The conclusions to be drawn from figure 4 are similar to those drawn from figure 3.
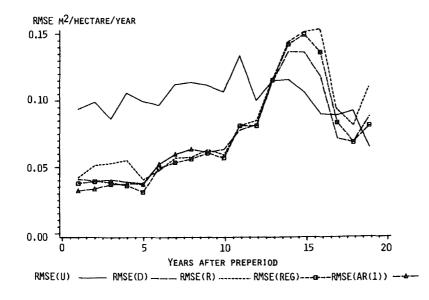

Figure 3. Data set no 2. *RMSE* of 5 estimators of treatment response. The preperiod is *10* years. (The curves are based upon 28 trials when year <11, 19 when year = 12, 11when year = 16, 6 when year = 18). *RMSE*(AR_{Tt}) is plotted only when $t<8$ because $AR_{Tt} \approx D_{Tt}$ when $t>8$, due to the mentioned convergence of *AR* series.)*Data sett nr. 2. RMSE for 5 respons estimatorer.*

Forrperioden er på 10 år (Kurvene er basert på 28 forsøk når år<11, 19 når år=12, 11 når år=16, 6 når år=18). RMSE(AR_{Tt}) er tegnet bare for t<8 fordi $AR_{Tt} \approx D_{Tt}$ når t>8, på grunn av konvergensen av AR tidsrekkerr.
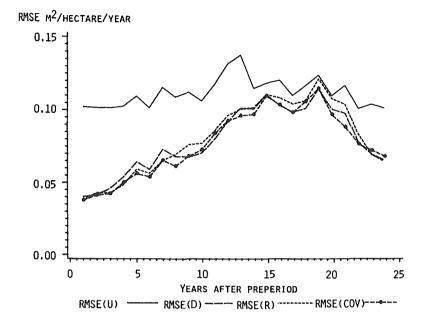
RMSE M²/HECTARE/YEAR

Figure 4. Data set no 3. *RMSE* of 4 estimators of treatment response. The preperiod is 5
   years. (The curves are based upon 107 trials when year <10, 90 when year = 15, 61
   when year = 20, 21 when year = 24). *Data sett nr. 3. RMSE for 4 respons
   estimatorer.*

   *Forrperioden er på 5 år. (Kurvene er basert på 107 forsøk når år<10, 61 når
   år=20, 21 når år=24)*

MØLLER & RYTTERSTEDT (1974), introduced the idea of "high" and "low"
quality of fertilization experiments based on difference in annual pretreatment
increment between the control and the fertilized plot. If a "high" quality experiment
is said to have a difference less than *30%*, then the data set no. 3 contains *65* of these
and *42* "low" quality trials. The *RMSEs* of U_{T_t}, D_{T_t} and R_{T_t} have been calculated
independently on those two groups. The *RMSE* of U_{T_t} from low quality experiments
is higher than above, about *0.15*. D_{T_t} and R_{T_t} do not seem to be influenced by this
distinction. U_{T_t} seems to be as good as D_{T_t} and R_{T_t} when the pretreatment
difference is less than *30%*.

Discussion and Conclusion

There is a significant advantage in using the simple adjusted estimators D_{Tt} or R_{Tt} compared to the unadjusted U_{Tt}, at least during the first *15* years of the trial. There was no further benefit in using complicated estimators like COV_{Tt}, REG_{Tt} or AR_{Tt}. As there was no difference between D_{Tt} or R_{Tt}, the distinction between absolute and relative increment changes is not reflected by these data. The increasing *RMSE*s of the adjusted estimators from *t=0* to *t=15* indicates the decreasing influence of preperiod conditions[6].

Based on *RMSE*, it is concluded that adjusted estimators are better than unadjusted estimators at least during the first *15* years of a trial. The adjusted estimators show very little difference as measured by mean square error. Thus the recommendation is to use the easily calculated D_{Tt} in comparable trials. This estimator also enables a short preperiod.

The conclusions based on uniformity trial data must (as shown above) be valid also in a real setting. However, the ranking given here is itself an estimate, thus another investigation with another natural variation may come up with another ranking of the estimators.

When fitting a treatment response function (using any of these response estimators) to treated data the error must exceed the one found here, which is due to environment, plot size etc. In addition there will be variation due to treatment, treatment*site interaction and "lack of fit" of the function. My experience with fertilization data indicates that a "first five year response function" typically has a variance of about *0.004* using the ratio estimator. The figures given here indicate a variance of about *0.002* (table 2). Thus about *50%* of the variation of a typical function is due to natural variation on the increment and the rest is due to sources connected with the treatment, including "lack of fit".

The ranking between U_{Tt}, R_{Tt} and COV_{Tt} does not agree with the one presented by LIPAS (1979), or *E3, E8* and *E5* in his notation. However LIPAS himself qualified his conclusions since true responses to fertilization were not known. This is a critical point and uniformity trials are the only way of overcoming this.

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[6] Figure 3 and 4 show an odd behavior when $t > \approx 18$. The results here are uncertain due to the low number of plots available for the calculations

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