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Estimation of birch substand parameters in order to divide a stand into two diameter classes

Estimering av parametre på delbestand av bjørk for å dele det opp i to diameterklasser

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Ås 1983

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Abstract

MØNNESS, E. N. 1983. Estimation of birch substand parameters in order to divide a stand into two diameter classes. (Estimering av parametre på delbestand av bjørk for å dele det opp i to diameterklasser.) Medd. Nor. inst. skogforsk. 38(8):1—35.

Functions are constructed to estimate stand parameters separately for birch trees with diameter greater than or less than 11 cm at breast height. The parameters are estimated when the parameters of the entire stand are known. The parameters in question are: Basal area mean diameter, Loreys height, volume, basal area, tree number (per hectare). The functions fulfil the functional relations that exist between these parameters. The functions are tested against an independent set of data.

Key words: Birch, stand parameter of substand.

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MØNNESS, E. N. 1983. Estimation of birch substand parameters in order to divide a stand into two diameter classes. (Estimering av parametre på delbestand av bjørk for å dele det opp i to diameterklasser.) Medd. Nor. inst. skogforsk. 38(8):1—35.

Det er laget funksjoner som estimerer bestandsparametre for bjørketrær med brysthøydediameter større eller mindre enn 11 cm, henholdsvis. Delbestandsparametrene estimeres når hele bestandets parametre er kjent. De aktuelle parametre er grunnflatemiddelstammens diameter, grunnflateveid middelhøyde, volum, grunnflate og treantall (pr. ha.). Funksjonene er bundet av de funksjonelle sammenhenger som gjelder mellom disse bestandsparametre. Funksjonene er testet mot et uavhengig materiale.

Nøkkelord: Bjørk, bestandsparametre for delbestand.

Preface

To enable prediction of economic value of birch forest the Department of Forest Economics of the Agricultural University of Norway have asked for this work to be done. The material was recorded by Division of Forest Management and Yield Studies, the Norwegian Forest Research Institute (NISK).

Helge Braastad (NISK) has given advice. Ida Svendsen (NISK) has typed the manuscript. Lars Strand (NISK) has read it. The English text was corrected by Sylvia Bredholt (the Agricultural University of Norway).

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Ås, February 1983

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I. Introduction

The Norwegian Forest Research Institute has shown an interest in yield from birch from the beginning of this century. We possess recordings from 50 permanent plots and 71 temporary plots of birch stands. Some plots have been thinned on several occasions. BRAASTAD (1967) has published yield tables for birch based upon this material where the yield of the entire stand is predicted. In order to make economic predictions more information is needed. The yield from trees with a diameter less than 11 cm at breast height is, for the time being, said to be of no value. This paper presents functions that calculate the stand parameters of trees having a diameter greater than 11 cm when the parameters of the entire stand are known. Together with yield tables (BRAASTAD, 1967) these functions may serve as basis for economic predictions. The yield tables make forecasts, these functions extend the result.

The functions have this special and limited purpose. A more general approach is to consider the diameter distribution of birch trees. Work of this kind has been done by VESTJORDET (1972) on *Picea abies* (L.) Karst., LUNDSTRØM (1981) on birch, and MØNNESS (1982) on *Pinus sylvestris* L.

The functions are to answer the following kind of questions: Given a birch stand by its stand parameters, how many trees have a diameter greater than 11 cm? What is their mean diameter, mean height and volume?

II. Material and methods

A. Material

The material consists of recordings of birch stands at different points in time. Each recording is considered here to be an independent observation. Some diagrams showing the composition and variation of the material is given in Appendix 2.

Each observation gives the following information:

The number of trees within 2 cm diameter classes.
The arithmetic mean diameter within the classes.
The arithmetic mean height of some trees within the classes.

(«Diameter» is always «diameter at breast height»). The recording is done separately on the thinning and the stand after thinning. The number of earlier thinned trees is also recorded. From these data the common stand parameters are calculated (Table 1). The material is divided at random into two parts which are called the estimation group and the testing group, respectively (Table 2). The functions are developed on the estimation group and are tested on the testing group giving measures of performance on future predictions.

The material consists of observations from the two species of birch, *Betula verrucosa* L. and *Betula pubescens* L. The calculations do not consider any difference between the species but they separate themselves by means of the site index.

A list of symbols is given in Appendix 1.

Table 1. Summary statistics of the material.

	Parameter	No. obs.	Mean	Standard Deviation	Minimum Value	Maximum Value
D_3	Basal areal mean diameter after thinning (cm)	361	14.56	5.98	4.50	31.30
D_{32}	d≥ 11 cm Basal area mean diameter (cm)	353	16.86	4.41	11.70	31.30
D_2	Basal area mean diam thinnings (cm)	264	11.92	5.62	2.98	28.08
D_{22}	d≥11 cm Basal area mean diam thinnings (cm)	221	15.75	3.62	11.00	28.08
G_3	Basal area after thinning (m^2 per hectare)	361	16.34	7.40	0.52	48.26
G_{32}	d≥11 cm Basal area (m^2 per hectare)	361	12.10	8.46	0.00	48.18
G_2	Basal area of thinnings (m^2 per hectare)	264	2.32	2.13	0.01	10.05
G_{22}	d≥11 cm Basal area thinnings (m^2 per hectare)	361	0.96	1.54	0.00	8.95
H_{40}	Site index Top height at $T_{1.3}$ = 40 year (m)	361	14.94	4.81	5.30	25.99
H_3	H (Lorey) after thinning (m)	361	14.53	5.14	4.97	26.96
H_{32}	d≥11 cm H (Lorey) (m)	353	15.23	4.69	6.70	26.96
H_2	H (Lorey) of thinned trees (m)	264	13.03	5.26	3.20	26.51
H_{22}	d≥11 cm H (Lorey) of thinned trees (m)	221	14.95	4.51	6.00	26.51
N_3	No.of standing trees after thinning per hectare	361	1532.48	1603.90	50.00	8532.81
N_{32}	d≥11 cm No. of standing trees per hectare	361	509.78	290.20	0.00	1519.59
N_2	No. of thinned trees per hectare	264	416.23	900.82	4.14	9908.61
N_{22}	d≥11 cm No. of thinned trees per hectare	361	46.61	68.82	0.00	396.56
$T_{1.3}$	Age at breast height (year)	361	48.71	18.17	12.00	94.00
V_3	Volume after thinning (m^3 per hectare)	361	113.67	75.32	3.87	502.62
V_{32}	d≥11 cm Volume (m^3 per hectare)	361	93.67	82.70	0.00	502.26
V_2	Volume of thinnings (m^3 per hectare)	264	14.30	15.25	0.03	80.31
V_{22}	d≥11 cm Volume of thinnings (m^3 per hectare)	361	7.34	13.34	0.00	80.31
	Area (hectare) of plot	361	0.16	0.17	0.02	2.66
	No of earlier thinned trees (per hectare)	361	2225.32	2604.04	0.00	16375.00
	No of times thinned	361	2.91	2.52	0.00	12.00

Table 2. Number of observations in each group.

	Estimation group	Testing group	Sum
Thinnings	191	73	264
Stand after thinning	258	103	361

B. A volume function

BRAASTAD (1966:47) suggested the following function for the volume of a single tree from its diameter (d) and height (h):

$$V(d,h) = -18.6827 + 2.1461*d^2 + 0.1283*d^2*h + 0.1380*d*h^2 - 0.6311*h^2 \quad (1)$$

Here (1) is taken for granted; each volume is calculated from this function. To calculate a stand volume from Dg and Hl, a slight modfication has to be done. From experience, the volume function yields too high a volume when used on Dg and Hl. It is therefore usual to reduce the Hl with a certain constant. That is, a constant α must be estimated such that $V(Dg, Hl*\alpha)*N$ gives the best fit to the volume calculated from single trees. This has been done on *Picea abies* (L.) Karst. (BRAASTAD 1975) and *Pinus sylvestris* L. (BRAASTAD 1980). In both papers α is estimated by a method which may be called «zero residual»: Select α such that

$$\Sigma(V_i - V(Dg_i, Hl_i * \alpha) * N_i) = 0 \quad (2)$$

where «i» is an index over stands. V_i is the volume calculated from single trees on stand no i.
Another procedure would be that of least squares:
Select α such that

$$\Sigma(V_i - V(Dg_i, Hl_i * \alpha) * N_i)^2 \quad (3)$$

is at its minimum.

Equation (2) is easily solved: It is a second degree polynomial in α. The least square method (3) is solved by a standard nonlinear regression procedure (SAS 1979). Estimates based upon the estimation data group are given in Table 3. The assymtotic standard error of the least squares method indicates a real difference between the two estimates. It is decided to use «zero residual» method to estimate α. Calculated on the entire material α equals 0.984. This value will be used. The testing material is still considered to be an independent set of data.

Table 3. Estimates of α.

	α	ass. std. error
«zero residual»	0.985195	—
«least squares»	0.98253	0.000965

C. A note on existence

With no trees of greater diameter than 11 cm, the volume, the tree number and the basal area of these «trees» are all zero, while Dg and Hl are not defined.

In the calculations all observations should have all stand parameters non-missing.

This is only a problem in the thinned material. Of 191 observations 31 observations have no trees with a diameter >11 cm. Omission of these observations from the material should not be done, they are important in predicting the behaviour of the functions at their lower boundaries. For these observations the Dg are defined to be 11 cm and the H1 are defined to be 10.4 m. The argument is as follows:

Consider a stand with a one-peaked diameter distribution. Suppose all diameters are continuously decreased. When the number of trees >11 cm are near to vanish their diameters are near 11 cm.

The argument on the H1 is more loosely. The H1 of trees with diameters >11 cm are regressed against Dg. H1=10.4 m is the estimated H1 at Dg=11cm. This pre-setting of Dg and H1 is not different from stating V, N and G to be zero. As the Dmax in the stand decreases below 11 cm, the volume (say) of trees >11 cm is really «zero» in a decreasing fashion, but remains to be zero.

D. Methods

The metod outlined in the introduction is used. The thinning and the stand after thinning are treated independently but equally. Estimates of stand parameters of trees with diameters > 11 cm do not rely on any kind of distribution function on diameters. What is gained is that each individual parameter is allowed to achieve its «optimal» prediction. What may be lost is «consistency» between the parameters. Given a known stand, calculate Dg, Hl, N, V and G. Then calculate these parameters among trees with diameters > 11 cm. Call this part 2 and index the parameters with a «2», thus the parameters are called Dg_2, Hl_2, $N._2$, $V._2$, $G._2$, respectively. Trees with diameters less than 11 cm are called part 1 and indexed correspondingly.

Functional relations between groups

The following relations will always hold:

$$V = V._1 + V._2$$
$$N = N._1 + N._2$$
$$G = G._1 + G._2$$
$$G * H1 = G._1 * Hl_1 + G._2 * Hl_2$$
$$Dg^2 * N = Dg_1{}^2 * N._1 + Dg_2{}^2 * N._2 \qquad (4)$$

Thus, given stand parameters and parameters of part 2, part 1 is also known.

It is reasonable that also the predictions should fulfil these restrictions. Thus, predictions are calculated on part 2. Part 1 are calculated as the difference due to (4). Some care is taken to avoid odd results.

Functional relations among the stand parameters

The relation

$$\frac{\pi}{4} \; Dg^2 * N = G \qquad (5)$$

is true on any set of trees. Also the relation

$$V = V(Dg, Hl * \alpha) * N \qquad (6)$$

is nearly true on any set of birch trees. ($V(\cdot)$ is the volume function (1)). Predictions should fulfil (5) and (6).

Transformations

Consider the prediction of $V._2$. The regression may be carried out on different transformations, e.g. on $V._2$, $V._2/V$ or $V._2—V$. They were all tried but no improvement was gained. No stand parameter is transformed.

Regression

The first stage is to establish a «best» regression function for each single part 2 parameter. As possible independent variables stand parameters of the entire stand and some transformations of them are used. The selection of independent variables has been done with a method called «maximum R improvement» (SAS, 1979). The Cp values of the different regression functions were also compared (GORMAN & TOMAN, 1966). The regression functions are given in Table 4. These functions are considered as the best attainable, when each parameter is considered separately.

A linear regression function of this form is not limited, but the following restrictions always prevail:

$$\begin{aligned} &Dg_2 > \max(11, Dg)\\ &Hl_2 > Hl\\ &0 < V._2 < V\\ &0 < N._2 < N\\ &0 < G._2 < G \end{aligned} \qquad (7)$$

That $Hl_2 > Hl$ is based on the assumption that height is nondecreasing when diameter grows. The other relations are consequences of definition. The estimates are forced to be restricted by (7) by truncation. This of course will improve their fit as measured in Table 4. The functions are not often «out of range». Thus, doing ordinary regression and truncating them afterwards does little harm to the properties of the estimates. A transformation, say a logistic, to eliminate the need of a truncation ought not to be used here. At a certain point, part 2 should equal the whole stand.

Table 4. «Best» unrestricted independent regression function on each dependent variable. Nearly all entries are significant at the 0.01 level.

Parameter	D_{32}	G_{32}	H_{32}	N_{32}	V_{32}
intercept	11.553	−11.511	0.2392	−511	49.172
D_3	0.5404	1.6464		33.506	
G_3	0.1114	0.5093		85.655	− 3.358
H_3		− 0.2656	1.1413	33.267	− 2.272
N_3		− 0.001285		− 0.156	
V_3		0.05008	0.003670	− 5.731	1.280
$D_3{*}D_3$	0.0176	− 0.03505	0.004836	− 1.857	− 0.106
$H_3{*}H_3$			−0.004338		
$\ln(N_3)$			0.3046		−14.405
$D_3{*}\ln(N_3)$	−0.0915		−0.04579		1.144
no. obs.	253	253	253	253	253
R^2	0.9796	0.9801	0.9975	0.8727	0.9916
$\sqrt{MSE}$	0.650	1.167	0.240	99	7.654
Parameter	D_{22}	G_{22}	H_{22}	N_{22}	V_{22}
intercept	8.677	1.053	−2.999	1.613	−8.196
G_3			−0.146		
H_3			0.507		
N_3	−0.000307	0.000244		0.0234	0.00138
$\ln(N_3)$			1.248	−24.597	
D_2				30.092	2.895
G_2					−6.886
H_2		−0.0714		− 4.447	−1.822
N_2					0.00438
V_2		0.0886		4.386	1.708
$D_2{*}D_2$	0.0261	−0.00266		− 0.938	−0.0643
$H_2{*}H_2$					0.0331
$\ln(N_2)$	0.7009	−0.5428	0.0176		
$D_2{*}\ln(N_2)$	−0.0272	0.0507			
no. obs.	191	191	191	191	191
R^2	0.9093	0.9519	0.9708	0.8389	0.9762
$\sqrt{MSE}$	1.156	.395	.768	31.5	2.49

Simultaneously predictions

The functions of Table 4 can not be used simultaneously as they do not fulfil the restrictions (5) and (6). This may be solved in different ways.

Only 3 of the 5 functions are used. The two remaining parameters are calculated from (5) and (6). In principle this could be done in 10 ways. (There is a problem in determining the adequate root of the second degree equation in some of these solutions). This scheme is tried in two versions.

The first, called method 1, uses the functions on Dg_2, Hl_2 and $N._2$ and then calculate $V._2$ and $G._2$ from these predictions. $V._2$ is calculated from (6) and (1) and $G._2$ from (5). The other, called method 2, uses the functions on Dg_2, Hl_2 and $V._2$. $N._2$ is then calculated according to (6) and $G._2$ according to (5). The first method will probably yield a poor fit to the volume. In both methods the calculated parameters are biased. This is so because an expectation may not be interchanged with a non-linear function. Anyway, the methods could yield acceptable estimates.

As method 3 a nonlinear multivariate regression method is used. The restrictions (5) and (6) are considered within the estimation process of the 5 functions: When predicting Dg and Hl (say) also the fit to $V = V(Dg, Hl*\alpha) * N$ is considered. This has been done with SAS/ETS (SAS/ETS 1980).

A stand with higher Dg_2 (say) than some model predict will probably also have a higher Hl_2. If the correlations across models are considered this would possibly decrease the prediction error. This is called «seemingly unrelated regression» (ZELLNER 1962) and is solved by a multistage least square method (DHRYMES 1970, SAS/ETS 1980). This kind of calculation was tried on all three methods above; method 1 and 2 were not improved while method 3 was.

Anyway method 2 turned out to be the best one and is the method to be used.

Simultaneously truncation

The model is to this stage as follows. Take the functions on Dg_2, Hl_2 and $V._2$ from Table 4. Calculate $G._2$ and $N._2$ from the predicted Dg_2, Hl_2 and $V._2$ by means of the relations (5) and (6). This yields estimates of the five part 2 parameters. They are the «best» estimates to observed data that also fulfil the relations (5) and (6).

The functions will, near their limits (7), probably reach their limits at different sets of the independent variables.

Firstly, consider the case when part 2 is near to vanish. That is, when nearly all trees have diameters less than 11 cm. If the predicted volume is zero, the basal area should vanish too. The functions reach their limits almost simultaneously. There is no harm to the functions to state the following rule on the estimates:

If $V._2 < 0$ then $V._2 = N._2 = G._2 = 0$
while Dg_2 and Hl_2 are undefined.
If $V._2 > 0$ then $Dg_2 = \max(11, Dg_2, Dg)$
and $Hl_2 = \max(Hl_2, Hl)$
$N._2$ and $G._2$ will then attain reasonable estimates.

Secondly, consider the case when part 2 is nearly equal to the entire stand. In this case the functions are more spread out. The volume may also here be used as the desision rule which is reasonable for part 2. However, it may sometimes produce some very odd trees for part 1. (Remember that part 1 is calculated as the difference (4) between the entire stand and the estimated part 2).

The following rule is stated:

If any of the parameters on part 2 reach their limit, estimate part 2 to be equal to the whole stand. This rule may bias the results «upwards», but the results from part 1 become more reasonable. The rule is used for about 10 % of the observations. The fit of the functions (all tree methods) is given Table 5.

Table 5. Bias and mean squares of the 3 estimation methods.

	D_{32}	G_{32}	H_{32}	N_{32}	V_{32}
$\sqrt{MSE}$ of table 4	0.650	1.167	0.240	99	7.65
Method 1					
Mean residual	−0.012	0.441	0.009	11.8	4.70
$\sqrt{MSE}$	0.661	2.065	0.238	88.9	19.5
no obs.	250	253	250	253	253
Method 2					
Mean residual	−0.043	−0.089	−0.030	−4.3	−0.103
$\sqrt{MSE}$	0.603	1.068	0.192	82	6.67
no obs.	240	253	240	253	253
Method 3					
Mean residual	0.004	0.400	−0.009	18.7	3.54
$\sqrt{MSE}$	0.850	1.633	0.259	110	12.0
no obs.	252	253	252	253	253
	D_{22}	G_{22}	H_{22}	N_{22}	V_{22}
$\sqrt{MSE}$ of table 4	1.156	0.395	0.768	31.5	2.49
Method 1					
Mean residual	0.101	0.057	0.013	−0.098	0.613
$\sqrt{MSE}$	1.110	0.405	0.655	26.8	3.30
no obs.	163	191	163	191	191
Method 2					
Mean residual	0.136	−0.051	0.006	−5.34	−0.275
$\sqrt{MSE}$	1.066	0.338	0.597	26.1	2.039
no obs.	161	191	161	191	191
Method 3					
Mean residual	−0.577	−0.022	−0.031	2.57	−0.059
$\sqrt{MSE}$	1.504	0.267	0.642	29.6	1.76
no obs.	171	191	171	191	191

III. Results

With a given set of stand parameters, the functions divide the stand in two parts: Trees with diameters greater or less than 11 cm. The Dg, Hl, V, N and G of the subgroups are calculated. The functions accept as input both measured and simulated stand parameters.

Thus, combined with yield tables for birch (BRAASTAD 1967) these functions predict birch stand development. Some examples of this use are given in Appendix 3. The functions are implemented in a computer program that also produce output tables.

The functions are tested against the independent testing material, results are given in Table 6. The predictions are highly correlated to the observed values. The observed and predicted values should, when plotted against each other, consentrate around the straight line with intercept zero and regression coefficient 1. Thus, consider the model observed = c + b * predicted + error. The hypothesis ($c = 0, \beta = 1$), that is equallity against linearity, is tested in Table 6. Also the mean square error (MSE) is given when equality is assumed.

Table 6. The functions tested against observed values on the testing material.
observed = $c+\beta*$ predicted.
«Equality» is tested against «linearity».

	D_{32}	G_{32}	H_{32}	N_{32}	V_{32}
c	−0.138	−0.393	−0.034	10.76	−1.56
β	1.017	1.027	1.001	0.944	1.018
no. obs.	93	103	93	103	103
R^2	0.944	0.9834	0.9975	0.9203	0.9922
$F(c=0, \beta=1)$	1.27	2.27	0.18	4.35	1.95
P(F)	0.28	0.11	0.83	0.02	0.15
$\sqrt{MSE}$	1.00	1.17	0.22	91.1	7.40

	D_{22}	G_{22}	H_{22}	N_{22}	V_{22}
c	1.41	0.018	−0.91	8.47	−0.05
β	0.930	0.928	1.054	0.744	0.954
no. obs.	57	73	57	73	73
R^2	0.8621	0.9012	0.9656	0.8257	0.9377
$F(c=0, \beta=1)$	3.15	2.90	2.82	23.9	2.18
P(F)	0.05	0.06	0.07	0.0001	0.12
$\sqrt{MSE}$	1.35	0.45	0.867	34.4	2.82

On stands after thinning only the test on N_{32} rejects the function. Thus on D_{32}, G_{32}, H_{32} and V_{32} the functions are an acceptable description of real observations. On N_{32} the predicted values are higher than observed. (As the tests are carried out on an independent set of data, «rejections» should be expected even if the functions are reasonable.)

On the thinnings the functions are more or less rejected but they will be used anyhow. Residual mean squares of all functions are also given in Table 6. These numbers indicate the error which is expected on future predictions

The functions are put into a computer program (written in SAS language) so that a table of parameters of part 1 and part 2 is produced for each input set of stand parameters. The program obey another condition: If Dg $>$ 30 cm or Hl $>$ 22 m then no trees get a diameter $<$ 11 cm. This makes the program work properly even outside the range of the material used here.

IV. Discussion

The functions have an acceptable performance on the testing group. Functional relations existing between their real counterparts are forced on the functions. Thus inner consistency overrules optimal statistical properties. Optimal statistical procedures that meet functional restrictions are tried but there was no gain as compared with easier methods on this material.

A comparison of mean square errors from Table 5 and 6, should be done. Table 5 shows the performance on the estimation material. Table 6 shows the performance on an independent set of data. The differences, of course positive, from Table 5 to Table 6, are small. Thus the functions should have an acceptable fit on future predictions. The expected errors can be read off from Table 6.

Among parameters on stand after thinning the N_{32} function was rejected on the testing material. When the functions are used one should therefore rely more on Dg_2, Hl_2, and $V._2$ and less on $N._2$. Anyway a direct estimation of $N._2$ would not produce estimates any better than this one (Table 5). Tree number is an unstable parameter. It is linearly dependent on the recording of very small trees, while the Dg, Hl and volume are less affected of small trees. The functions on thinnings are less reliable. I have compared the results visually on each observation and they are quite reasonable.

Summary

Diagrams to show the composition of the material are given in Appendix 2. The material is divided in two parts, the estimation and the testing group, respectively. The theory is developed on the estimation data group. The validity of the results is tested against the independent testing data group.

A birch is said to have an economical value when its diameter is greater than 11 cm. Norwegian yield tables on birch simulate stand development as given by Dg, Hl, N, V and G. These parameters are here input to functions that output the same parameters on trees greater than 11 cm. The stand parameters of any set of trees are functionaly dependent. Parameters on trees with diameters less than or greater than 11 cm add up to the parameters of the entire stand and they are nonnegative. G is given by Dg and N, V is (nearly) given by Dg, Hl and N. All these restrictions are considered in the estimation. An easy step-by-step method is preferred because no gain was achieved by advanced nonlinear multivariate techniques. The performance of the functions is given in Table 6. The functions are to be used in connection with birch yield tables. The output from a stand simulation is distributed in two diameter classes by the function. Some examples of this use are given in Appendix 3.

Estimering av parametre på delbestand av bjørk for å dele det opp i to diameterklasser

En bjørk er for tiden ansett økonomisk drivverdig om dens brysthøydediameter er større enn 11 cm. Det er med dagens norske produksjonstabeller ikke mulig å foreta en slik oppdeling etter diameter. Hensikten med dette arbeidet er å utvikle funksjoner som gjør det. Basert på bestandsparametre for hele bestandet estimeres «bestandsparametre» klassevis for trær større eller mindre enn 11 cm. Beregningene baserer seg på forsøksfelt i bjørk lagt ut og vedlikeholdt av avdeling for skogbehandling og skogproduksjon, NISK.

Det er både faste felt og engangsfelt. Endel felt er fulgt gjennom flere revisjoner og tynninger. Hver revisjon behandles som en uavhengig observasjon og tynninger behandles separat. Middeltall for materialet er gitt i tabell 1 og det er plottet i appendix 2. Materialet er delt i to: Teorien utvikles på den ene del, mens en kontrollert test utføres på det uavhengige test-materialet (tabell 2).

Et rett-frem teknikk er brukt: Gitt bestandsdata, hva er Dg for trær $>$ 11 cm? Dette er løst ved regresjon for hver parameter. Nå er det en rekke funksjonelle sammenhenger som alltid gjelder blant bestandsparametre. Likning (5) gjelder bestandig og (6) er en rimelig sammenheng. Bestandsparametre for hele bestandet og for trær $>$ 11 cm vil alltid oppfylle (7). Dessuten vil hele bestandet, sammen med trær $>$ 11 cm og $<$ 11 cm alltid oppfylle (4). Funksjonene bør også oppfylle disse krav.

Ligningene (4) sier at trær $<$ 11 cm og $>$ 11 cm skal summere seg eksakt opp til hele bestandet. Dette er løst ved at det kun beregnes funksjoner for trær > 11 cm. Deretter finnes resten ved ren subtraksjon (4). «Differansen» vil ikke alltid bli forventningsrett og den kan skape noen urimelige trær. Dette må taes hensyn til senere i beregningene.

Funksjoner for Dg_2, Hl_2, $N._2$, $V._2$ og $G._2$ beregnes uavhengig av hverandre, med Dg, Hl, N, V og G som uavhengige variable (Det inngår også noen transformasjoner og endel andre variable er også forsøkt). Disse funksjoner er gitt i tabell 4. For å møte kravet (5) og (6) forsøkes 3 metoder. Metode 1 nytter funksjonene for Dg_2, Hl_2 og $N._2$ og beregner $V._2$ og $G._2$ ut ifra dem ved (5) og (6).

Metode 2 nytter funksjonene for Dg_2, Hl_2 og $V._2$ og beregner så $N._2$ og $G._2$ ut ifra (5) og (6). For begge disse metoder gjelder at de beregnede parametre vil ha kompliserte statistiske egenskaper og de vil ikke være forventningsrette (En kan ikke flytte forventning igjennom de ikke-lineære relasjoner (5) og (6)). I begge tilfelle kan estimatene av «grunnfunksjonene» teoretisk sett forbedres ved å ta hensyn til restriksjonene (5) og (6) under selve estimeringen. (Simultan ikke-lineær multivariat regresjon). En skulle kunne vente bedre estimater med denne metode. På dette materiale skjedde ikke det, metode 2 vil bli brukt. For å tilfredsstille kravet (7) blir hver enkelt funksjon avstumpet til sin grense om nødvendig. Dersom en avstumper en parameter og ikke en annen, vil «differanse-bestandet» trær < 11 cm, kunne få urimelig sammensatte bestandsparametre. Dette er håndtert på følgende måte:

Når $V._2$ estimeres til å bli null, settes hele bestandet til å bestå av trær < 11 cm.

Når nesten alle trær er > 11 settes følgende regel: Så sant en parameter overskrider grenseverdien settes alle trær til å ha diameter > 11 cm. Dette vil vri estimatene litt oppover, men en unngår en tabell med urimelige småtrær (urimelige d/h/v forhold). Dette er gjort for alle 3 metoder og resultatene er gitt i tabell 5. Metode 2 er ansett best av metodene. Denne er så testet mot testmaterialet (tabell 6). Funksjonene kan brukes både med målte og simulerte inngangsverdier. Eksempler på det siste er gitt i appendix 3. Sammen med produksjonstabeller for bjørk fremskrives en to-klassevis diameterfordeling av tenkte bestand.

til kovarians imellom dem. Dette gav ingen målbar forbedring i dette materialet. Metode 3 er en metode som tar hensyn

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Appendix 1

D, Dg — Basal area mean diameter (cm)
Grunnflatemiddelstammens diameter (cm)

G — Basal area (m^2 per hectare)
Grunnflate, m^2 pr. ha

H, Hl — Loreys height (m)
Grunnflateveid middelhøyde (m)

H_0 — Top height, arithmetic mean height of 100 largest trees per hectare
Overhøyde, aritmetisk middel av de 100 grøvste trær pr. ha.

H_{40} — Site index. H_0 at $T_{1.3} = 40$
Bonitet H_0 *ved* $T_{1.3} = 40$.

MSE — Mean square error
Midlere kvadratavvik

N — Number of trees per hectare
Treantall pr. ha

$T_{1.3}$ — Age at breast height (year)
Alder i brysthøyde (år)

V — Volume, m^3 per hectare
Volume, m^3 pr. ha.

Indexing on stand parameters.

The symbols D, G, H, N, V are doubly indexed, the volume is written out in detail.

V_3	volume after thinning
V_2	volume of thinnings
V_{32}	volume of trees with diam. > 11 cm after thinning
V_{31}	volume of trees with diam. < 11 cm after thinning.
V_{22} and V_{21}	correspondingly
V	volume of either thinnings or after thinning
$V_{.2}$	volume of trees with diam. > 11 cm, either thinnings or after thinning
$V_{.1}$	correspondingly.

D, G, H, N are ondexed correspondingly. D and H are, due to tradition, indexed Dg and H1, respectively.

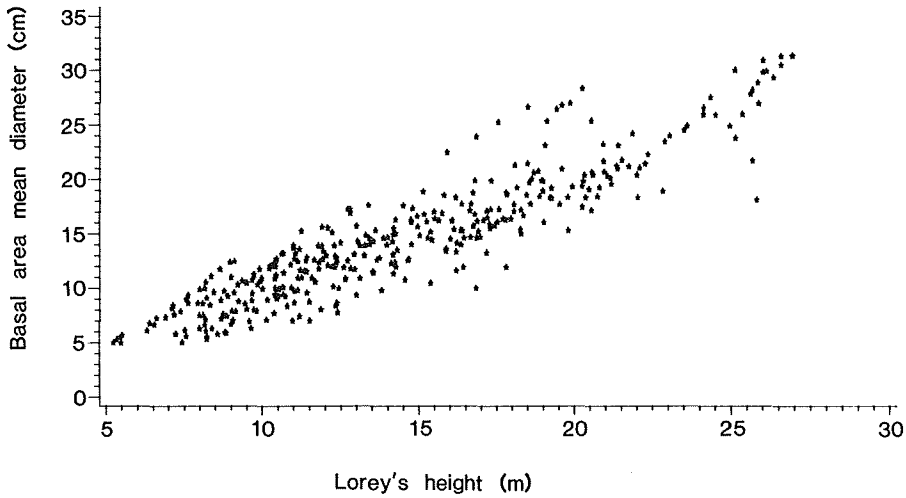
Symbolene D, G, H, N, V er dobbelt indeksert.
Volumet tjener som eksempel.

V_3	*volum etter tynning*
V_2	*volum av tynning*
V_{32}	*volum av trær med diam. > 11 cm etter tynning*
V_{31}	*volum av trær med diam. < 11 cm etter tynning*
V_{22} og V_{21}	*tilsvarende*
V	*volum, enten* V_3 *eller* V_2
$V_{.2}$	*volum, enten* V_{32} *eller* V_{22}
V_1	*tilsvarende.*

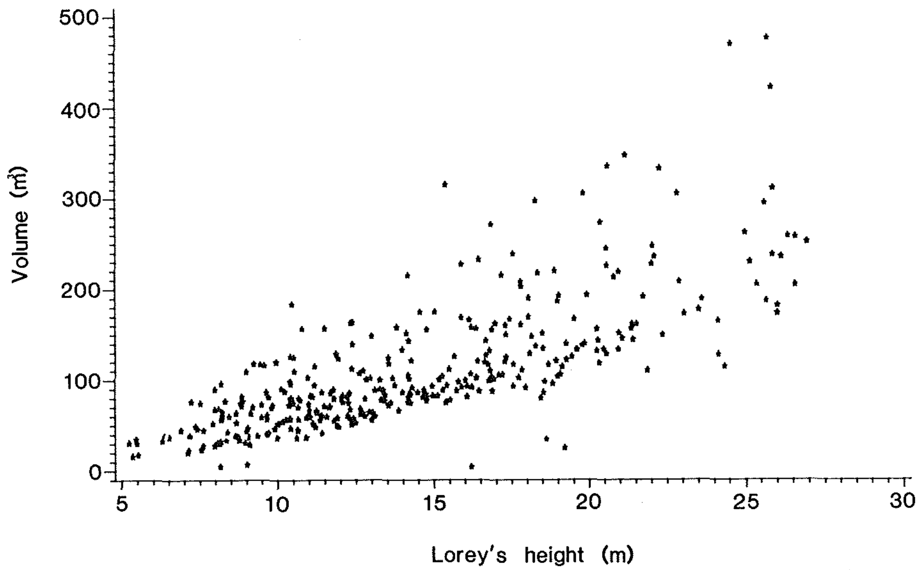
D, H, G, N *og* V *er indeksert slik. Pga. tradisjon skrives* Dg *og* Hl *istedet for* D *og* H.

Appendix 2

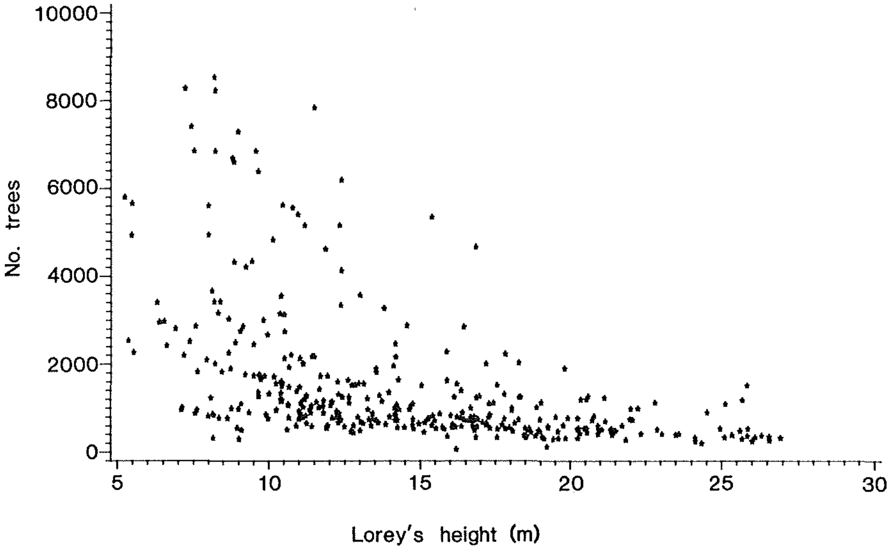
To show how the material is composed, some computer made diagrams between some stand parameters are presented.



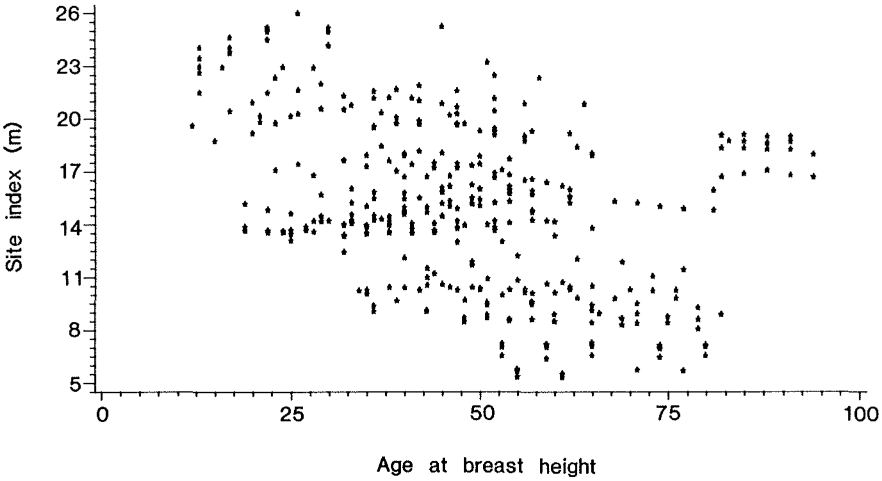
Diameter against height, stand after thinning



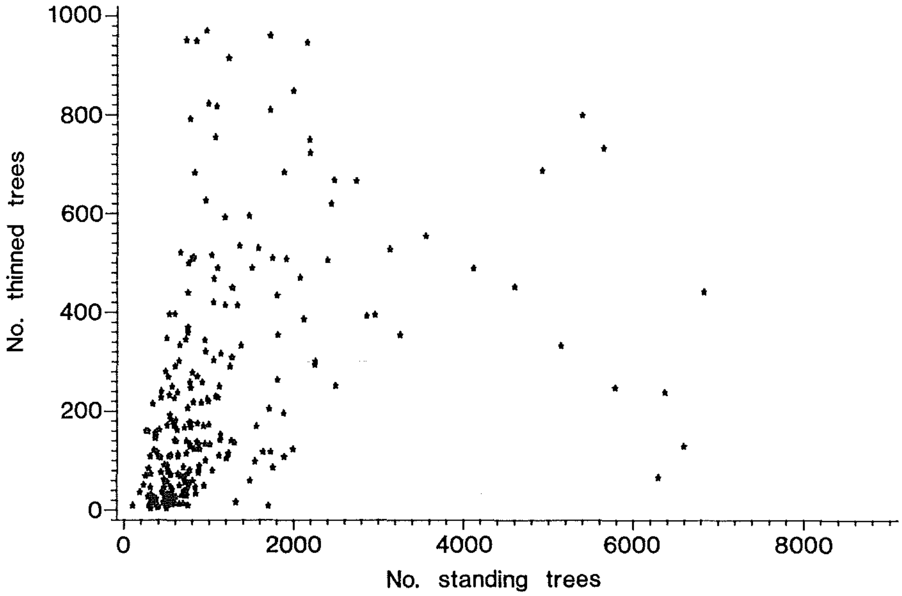
Volume of standing trees per hectare against height



Number of standing trees per hectare against height

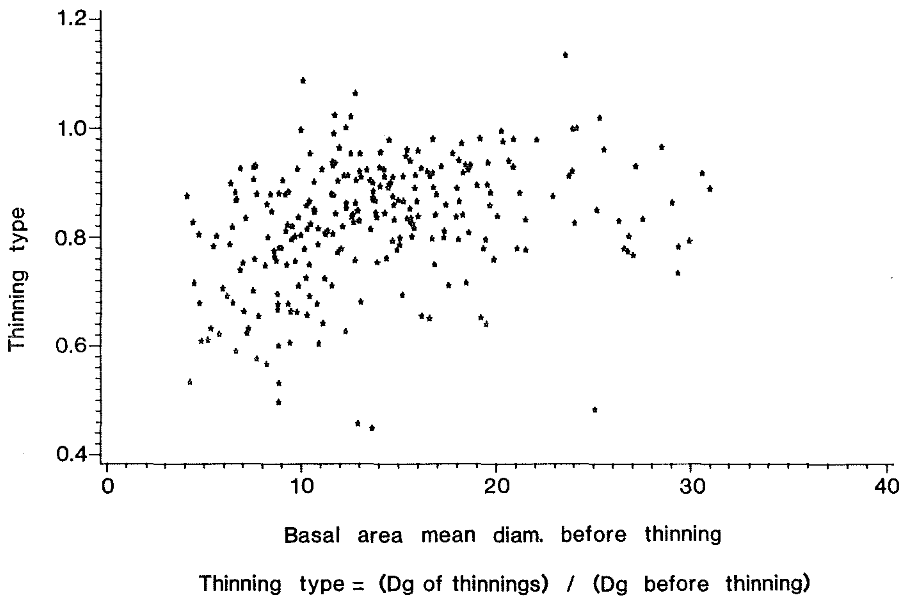


Site index against age



Per hectare. (Thinnings above 1000 omitted)

Number of thinned trees against number standing trees



Thinning type = (Dg of thinnings) / (Dg before thinning)

Thinning type

Appendix 3

This appendix shows some yield tables for birch together with the estimated substands parameters using the present functions. The yield tables are improvements of those presented in BRAASTAD (1967). He has improved the height/diameter relation and the loss of trees due to «self thinning» are implemented. The tree numbers therefore decrease with age. The new tables are yet not published elsewhere.

TABLE /A

BETULA PUBESCENS

H40= 8.0 1 THINNING PROGRAMME 1 PER HECTARE

AGE			BEFORE THINNING					THINNINGS				AFTER THINNING					TOTAL PROD.	MAI	C.A.I.		
TT	TBH	HO	H	D	N	G	V	H	D	N	V	H	D	N	G	V	VT	V	D	G	V
		M	M	CM		M2	M3	M	CM		M3	M	CM		M2	M3	M3	M3	MM	M2	M3
39	30	7.0	6.0	7.0	2500	9.7	30	5.4	5.3	900	6	6.2	7.8	1600	7.7	24	30	0.8			
																			1.7	C.3	1.5
44	35	7.5	6.7	8.7	1600	9.4	31	5.9	6.1	31	0	6.7	8.7	1569	9.3	31	37	0.8			
																			1.4	C.3	1.6
49	40	8.0	7.2	9.4	1569	10.9	39	6.4	6.6	31	0	7.2	9.5	1538	10.8	39	45	0.9			
																			1.4	0.3	1.7
54	45	8.5	7.7	10.2	1538	12.4	47	6.8	7.1	30	0	7.7	10.2	1508	12.3	47	53	1.0			
																			1.4	0.4	1.9
59	50	8.9	8.2	10.9	1508	14.1	56	7.2	7.6	30	0	8.2	11.0	1478	13.9	56	62	1.1			
																			1.3	0.3	2.0
64	55	9.3	8.6	11.6	1478	15.6	66	7.6	8.1	29	1	8.7	11.7	1449	15.5	65	72	1.1			
																			1.2	0.3	2.0
69	60	9.8	9.1	12.2	1449	17.0	75	8.0	8.6	28	0	9.1	12.3	1421	16.9	75	82	1.2			
																			1.0	C.3	2.0
79	70	10.5	9.9	13.3	1421	19.8	95	8.7	9.3	56	2	9.9	13.5	1365	19.4	93	102	1.3			
																			0.8	-0.2	1.8
89	80	11.3	10.6	14.3	1365	21.8	111	9.3	10.0	54	1	10.6	14.4	1311	21.4	110	120	1.3			
																			0.6	C.2	1.5
99	90	11.9	11.2	15.0	1311	23.2	125	9.9	10.5	52	2	11.3	15.2	1259	22.8	123	135	1.4			
																			0.4	C.1	1.2
109	100	12.6	11.8	15.6	1259	24.0	134	10.5	10.9	50	2	11.8	15.8	1209	23.6	132	146	1.3			
																			0.3	C.1	C.9
119	110	13.2	12.4	16.1	1209	24.5	141	11.0	11.2	48	2	12.4	16.2	1161	24.0	139	155	1.3			

TABELL / B TREE SIZE DISTRIBUTION Betula pubescens per hectare
DIMENSJONSFORDELING BJØRK pr. hektar
H40 = 8

t1.3	Ho		THINNINGS / TYNNING diameter (cm) < 11	>= 11	sum	AFTER THINNING / ETTER TYNNING diameter (cm) < 11	>= 11	sum
		Dg	5.3		5.3	7.8		7.8
30	7.0	Hl	5.4		5.4	6.2		6.2
		N	900	0	900	1600	0	1600
		V	6.0	0.0	6.0	24.0	0.0	24.0
		Dg				10.9	15.3	13.5
70	10.5	Hl				9.5	10.1	9.9
		N				620	745	1365
		V				23.8	69.2	93.0
		Dg				11.0	15.9	14.4
80	11.3	Hl				10.2	10.7	10.6
		N				463	848	1311
		V				21.9	88.1	110.0
		Dg				11.0	16.4	15.2
90	11.9	Hl				11.0	11.4	11.3
		N				374	885	1259
		V				20.8	102.2	123.0
		Dg				11.0	16.9	15.8
100	12.6	Hl				11.7	11.8	11.8
		N				305	904	1209
		V				19.4	112.6	132.0
		Dg				11.0	17.1	16.2
110	13.2	Hl				12.3	12.4	12.4
		N				250	911	1161
		V				17.8	121.2	139.0

TABLE 2 A BETULA PUBESCENS

H40=11.0 2 THINNINGS PROGRAMME 1 PER HECTARE

AGE			BEFORE THINNING					THINNINGS				AFTER THINNING					TOTAL PROD.	MAI	C.A.I.		
TT	TBH	HO	H	D	N	G	V	H	D	N	V	H	D	N	G	V	VT	V	D	G	V
		M	M	CM		M2	M3	M	CM		M3	M	CM		M2	M3	M3	M3	MM	M2	M3
24	17	7.1	6.2	7.0	2500	9.6	31	5.5	5.3	900	7	6.4	7.8	1600	7.6	24	31	1.3			
																			2.6	0.5	2.6
27	20	7.7	6.9	8.6	1600	9.2	32	6.1	6.0	19	0	6.9	8.6	1581	9.2	32	39	1.4			
																			2.4	0.5	2.9
32	25	8.6	7.9	9.8	1581	11.9	46	6.9	6.8	31	0	7.9	9.8	1550	11.8	46	53	1.7			
																			2.1	0.5	3.2
37	30	9.4	8.7	10.9	1550	14.4	62	7.7	7.6	30	1	8.7	10.9	1520	14.3	61	69	1.9			
																			1.9	0.5	3.4
42	35	10.2	9.5	11.9	1520	16.9	78	8.5	8.3	30	0	9.5	12.0	1490	16.7	78	86	2.0			
																			1.7	0.5	3.5
47	40	11.0	10.3	12.8	1490	19.2	95	9.2	9.0	29	1	10.3	12.9	1461	19.0	94	103	2.2			
																			1.5	0.5	3.5
52	45	11.7	11.0	13.6	1461	21.3	112	10.3	11.0	700	32	11.3	15.6	761	14.6	80	121	2.3			
																			1.6	0.3	2.6
57	50	12.4	12.0	16.5	761	16.2	92	10.7	11.5	15	0	12.0	16.5	746	16.0	92	133	2.3			
																			1.2	0.2	2.1
62	55	13.1	12.7	17.1	746	17.2	102	11.3	12.0	14	1	12.7	17.2	732	17.1	101	143	2.3			
																			1.2	0.3	2.2
67	60	13.7	13.3	17.8	732	18.3	112	11.9	12.5	14	1	13.3	17.9	718	18.1	111	154	2.3			
																			1.5	0.3	2.6
77	70	14.8	14.5	19.4	718	21.3	138	12.9	13.6	28	3	14.5	19.6	690	20.9	135	181	2.4			
																			1.2	0.3	2.2
87	80	15.9	15.5	20.8	690	23.5	158	13.9	14.6	27	3	15.5	21.0	663	23.0	155	204	2.3			
																			0.9	0.2	1.7
97	90	16.8	16.4	21.9	663	25.0	172	14.7	15.3	26	3	16.5	22.1	637	24.5	169	221	2.3			
																			0.7	0.1	1.3

TABELL 2 B TREE SIZE DISTRIBUTION Betula Pubescens per hectare
DIMENSJONSFORDELING BJØRK pr. hektar
H40 = 11

			THINNINGS TYNNING diameter (cm)			AFTER THINNING ETTER TYNNING diameter (cm)		
t1.3	Ho		< 11	>= 11	sum	< 11	>= 11	sum
		Dg	5.1	12.7	5.3	7.8		7.8
17	7.1	Hl	5.2	8.9	5.5	6.4		6.4
		N	887	13	900	1600	0	1600
		V	6.2	0.8	7.0	24.0	0.0	24.0
		Dg	9.8	14.2	11.0	9.4	16.4	15.6
45	11.7	Hl	10.0	10.8	10.3	10.7	11.3	11.3
		N	539	161	700	109	652	761
		V	18.4	13.6	32.0	5.2	74.8	80.0
		Dg					17.2	17.2
55	13.1	Hl					12.7	12.7
		N				0	732	732
		V				0.0	101.0	101.0
		Dg					19.6	19.6
70	14.8	Hl					14.5	14.5
		N				0	690	690
		V				0.0	135.0	135.0
		Dg					21.0	21.0
80	15.9	Hl					15.5	15.5
		N				0	663	663
		V				0.0	155.0	155.0
		Dg					22.1	22.1
90	16.8	Hl					16.5	16.5
		N				0	637	637
		V				0.0	169.0	169.0

TABLE 3A BETULA PUBESCENS

H40=14.0 2 THINNINGS PROGRAMME 1 PER HECTARE

AGE			BEFORE THINNING					THINNINGS				AFTER THINNING					TOTAL PROD.	MAI	C.A.I.		
TT	TBH	HO	H	D	N	G	V	H	D	N	V	H	D	N	G	V	VT	V	D	G	V
		M	M	CM		M2	M3	M	CM		M3	M	CM		M2	M3	M3	M3	MM	M2	M3
16	10	6.8	5.9	6.9	2500	9.2	28	5.3	5.2	900	6	6.1	7.6	1600	7.3	22	28	1.8			
																			3.6	0.8	4.0
21	15	8.2	7.5	9.5	1600	11.3	42	6.6	6.6	31	0	7.5	9.5	1569	11.1	42	48	2.3			
																			3.1	0.8	4.9
26	20	9.5	8.9	11.1	1569	15.1	66	7.9	7.8	31	1	8.9	11.1	1538	15.0	65	72	2.8			
																			2.7	0.8	5.3
31	25	10.8	10.1	12.5	1538	18.8	92	9.0	8.7	30	1	10.1	12.5	1508	18.6	91	99	3.2			
																			2.3	0.7	5.5
36	30	11.9	11.3	13.7	1508	22.3	119	10.5	11.1	700	34	11.6	15.6	808	15.5	85	127	3.5			
																			2.6	0.5	4.4
41	35	13.0	12.7	16.9	808	18.2	107	11.3	11.8	16	0	12.7	17.0	792	18.0	107	149	3.6			
																			2.1	0.5	4.0
46	40	14.0	13.7	18.1	792	20.3	127	12.2	12.6	15	1	13.7	18.2	777	20.1	126	169	3.7			
																			1.9	0.4	3.8
51	45	14.9	14.6	19.1	777	22.3	145	13.1	13.4	15	2	14.6	19.2	762	22.1	143	188	3.7			
																			1.9	0.4	3.9
56	50	15.8	15.5	20.2	762	24.3	163	13.9	14.1	15	2	15.5	20.3	747	24.1	161	208	3.7			
																			1.7	0.4	3.5
61	55	16.6	16.3	21.1	747	26.1	179	14.6	14.8	14	2	16.3	21.2	733	25.9	177	226	3.7			
																			1.5	0.4	3.2
66	60	17.4	17.1	21.9	733	27.7	193	15.3	15.4	14	2	17.1	22.0	719	27.4	191	242	3.7			
																			1.3	0.3	2.8
76	70	18.8	18.4	23.3	719	30.7	219	16.6	16.3	28	4	18.5	23.6	691	30.1	215	270	3.6			
																			0.9	0.2	2.2

TABELL 3B TREE SIZE DISTRIBUTION Betula pubescens per hectare
DIMENSJONSFORDELING BJØRK pr. hektar
H40 = 14

			THINNINGS TYNNING diameter (cm)			AFTER THINNING ETTER TYNNING diameter (cm)		
t1.3	Ho		< 11	>= 11	sum	< 11	>= 11	sum
10	6.8	Lg	5.2		5.2	7.6		7.6
		Hl	5.3		5.3	6.1		6.1
		N	900	0	900	1600	0	1600
		V	6.0	0.0	6.0	22.0	0.0	22.0
30	11.9	Lg	9.7	14.3	11.1	11.0	16.4	15.6
		Hl	10.2	10.9	10.5	11.2	11.6	11.6
		N	511	189	700	144	664	808
		V	17.9	16.1	34.0	7.2	77.8	85.0
40	14.0	Lg					18.2	18.2
		Hl					13.7	13.7
		N				0	777	777
		V				0.0	126.0	126.0
50	15.8	Lg					20.3	20.3
		Hl					15.5	15.5
		N				0	747	747
		V				0.0	161.0	161.0
60	17.4	Lg					22.0	22.0
		Hl					17.1	17.1
		N				0	719	719
		V				0.0	191.0	191.0
70	18.8	Lg					23.6	23.6
		Hl					18.5	18.5
		N				0	691	691
		V				0.0	215.0	215.0

TABLE 4A

BETULA VERRUCOSA

H40=17.0 3 THINNINGS PROGRAMME 1 PER HECTARE

AGE			BEFORE THINNING					THINNINGS				AFTER THINNING					TOTAL PROD.	MAI	C.A.I.		
TT	TBH	HO	H	D	N	G	V	H	D	N	V	H	D	N	G	V	VT	V	D	G	V
		M	M	CM		M2	M3	M	CM		M3	M	CM		M2	M3	M3	M3	MM	M2	M3
20	15	9.8	8.6	7.6	2500	11.4	50					8.6	7.6	2500	11.4	50	50	2.5			
																			3.2	1.0	6.4
23	18	10.8	9.6	8.6	2500	14.5	69	8.7	6.0	29	0	9.6	8.6	2471	14.4	69	69	3.0			
																			2.9	1.0	7.2
27	22	12.2	10.9	9.8	2471	18.6	98	10.1	7.6	1200	29	11.2	11.5	1271	13.1	69	98	3.6			
																			3.4	0.8	6.1
30	25	13.1	12.1	12.5	1271	15.6	88	11.0	8.7	15	1	12.1	12.5	1256	15.5	87	117	3.9			
																			3.0	0.8	6.5
35	30	14.5	13.5	14.0	1256	19.4	120	12.3	9.8	25	1	13.5	14.1	1231	19.2	119	150	4.3			
																			2.7	0.8	6.9
40	35	15.8	14.8	15.4	1231	23.0	153	14.2	12.7	500	41	15.1	17.0	731	16.7	112	184	4.6			
																			3.1	0.6	6.1
45	40	17.0	16.3	18.6	731	19.9	143	14.9	13.0	14	2	16.3	18.7	717	19.7	141	215	4.8			
																			2.7	0.6	6.0
50	45	18.1	17.5	20.0	717	22.6	171	15.9	14.0	14	1	17.5	20.1	703	22.4	170	245	4.9			
																			2.5	0.6	6.2
55	50	19.1	18.5	21.4	703	25.3	200	16.8	15.0	14	1	18.5	21.5	689	25.1	199	275	5.0			
																			2.6	0.6	6.7
60	55	20.1	19.5	22.8	689	28.2	232	18.9	20.1	300	78	19.8	24.7	389	18.6	154	308	5.1			
																			3.3	0.5	5.3
65	60	20.9	20.7	26.3	389	21.2	180	18.7	18.4	7	1	20.8	26.4	382	21.0	179	334	5.1			
																			2.8	0.4	4.8
70	65	21.7	21.6	27.8	382	23.2	203	19.5	19.5	7	2	21.6	27.9	375	23.0	201	358	5.1			
																			2.8	0.5	4.9

TABELL 4B TREE SIZE DISTRIBUTION Betula Verrucosa per hectare
DIMENSJONSFORDELING BJØRK pr. hektar
H40 = 17

t1.3	Ho		THINNINGS / TYNNING diameter (cm) < 11	>= 11	sum	AFTER THINNING / ETTER TYNNING diameter (cm) < 11	>= 11	sum
		Dg	6.0	13.3	7.6	9.3	14.0	11.5
.22	12.2	Hl	8.9	11.5	10.1	10.3	11.8	11.2
		N	1017	183	1200	751	520	1271
		V	14.9	14.1	29.0	23.6	45.4	69.0
		Dg		12.7	12.7	11.0	17.4	17.0
35	15.8	Hl		14.2	14.2	13.9	15.2	15.1
		N	0	500	500	71	660	731
		V	0.0	41.0	41.0	5.8	106.2	112.0
		Dg		20.1	20.1		24.7	24.7
55	20.1	Hl		18.9	18.9		19.8	19.8
		N	0	300	300	0	389	389
		V	0.0	78.0	78.0	0.0	154.0	154.0
		Dg					26.8	26.8
60	20.9	Hl					20.8	20.8
		N				0	382	382
		V				0.0	179.0	179.0
		Dg					27.9	27.9
65	21.7	Hl					21.6	21.6
		N				0	375	375
		V				0.0	201.0	201.0

TABLE 5 A BETULA VERRUCOSA

H40=20.0 3 THINNINGS PROGRAMME 1 PER HECTARE

AGE			BEFORE THINNING					THINNINGS				AFTER THINNING					TOTAL PROD.	MAI	C.A.I.		
TT	TBH	HO	H	D	N	G	V	H	D	N	V	H	D	N	G	V	VT	V	D	G	V
		M	M	CM		M2	M3	M	CM		M3	M	CM		M2	M3	M3	M3	MM	M2	M3
17	12	10.2	8.9	7.7	2500	11.7	53					8.9	7.7	2500	11.7	53	53	3.1			
																			4.2	1.4	9.2
21	16	12.0	10.7	9.4	2500	17.3	90	10.0	7.3	1200	26	11.0	11.0	1300	12.3	64	90	4.3			
																			4.4	1.1	8.4
26	21	14.0	13.0	13.2	1300	17.7	106	11.9	9.2	25	1	13.0	13.2	1275	17.6	105	132	5.1			
																			3.7	1.0	9.3
31	26	15.8	14.8	15.1	1275	22.8	152	14.2	12.4	500	39	15.1	16.6	775	16.8	113	179	5.8			
																			3.8	0.8	7.9
35	30	17.1	16.4	18.1	775	20.0	144	15.0	12.7	12	1	16.4	18.2	763	19.9	143	210	6.0			
																			3.3	0.8	7.9
40	35	18.6	17.9	19.9	763	23.7	183	16.4	13.9	15	2	17.9	20.0	748	23.4	181	250	6.3			
																			2.9	0.7	7.9
45	40	20.0	19.3	21.4	748	26.9	221	18.7	18.7	300	67	19.6	23.1	448	18.7	154	290	6.4			
																			3.5	0.6	6.5
50	45	21.2	20.8	24.8	448	21.7	187	18.9	17.4	8	2	20.9	24.9	440	21.5	185	323	6.5			
																			2.9	0.5	6.0
55	50	22.3	22.0	26.4	440	24.1	216	20.0	18.5	8	2	22.0	26.5	432	23.9	214	354	6.4			
																			2.7	0.5	5.8
60	55	23.4	23.1	27.9	432	26.4	243	20.9	19.5	8	2	23.1	28.0	424	26.1	241	383	6.4			
																			2.7	0.5	5.9

TABELL 5B TREE SIZE DISTRIBUTION Betula Verrucosa per hectare
DIMENSJONSFORDELING BJØRK pr. hektar
H40 = 20

t1.3	Ho		THINNINGS / TYNNING diameter (cm) < 11	>= 11	sum	AFTER THINNING / ETTER TYNNING diameter (cm) < 11	>= 11	sum
		Dg	6.0	13.2	7.3	9.0	13.8	11.0
16	12.0	Hl	9.0	11.5	10.0	10.1	11.7	11.0
		N	1050	150	1200	829	471	1300
		V	14.5	11.5	26.0	24.6	39.4	64.0
		Dg		12.4	12.4	11.0	17.1	16.6
26	15.8	Hl		14.2	14.2	13.8	15.2	15.1
		N	0	500	500	100	675	775
		V	0.0	39.0	39.0	7.5	105.5	113.0
		Dg		18.7	18.7		23.1	23.1
40	20.0	Hl		18.7	18.7		19.6	19.6
		N	0	300	300	0	448	448
		V	0.0	67.0	67.0	0.0	154.0	154.0
		Dg					24.9	24.9
45	21.2	Hl					20.9	20.9
		N				0	440	440
		V				0.0	185.0	185.0
		Dg					26.5	26.5
50	22.3	Hl					22.0	22.0
		N				0	432	432
		V				0.0	214.0	214.0
		Dg					28.0	28.0
55	23.4	Hl					23.1	23.1
		N				0	424	424
		V				0.0	241.0	241.0

TABLE 6 A

BETULA VERRUCOSA

H40=23.0 3 THINNINGS PROGRAMME 1 PER HECTARE

AGE TT	AGE TBH	HO M	BEFORE THINNING H M	BEFORE THINNING D CM	BEFORE THINNING N	BEFORE THINNING G M2	BEFORE THINNING V M3	THINNINGS H M	THINNINGS D CM	THINNINGS N	THINNINGS V M3	AFTER THINNING H M	AFTER THINNING D CM	AFTER THINNING N	AFTER THINNING G M2	AFTER THINNING V M3	TOTAL PROD. VT M3	MAI V M3	C.A.I. D MM	C.A.I. G M2	C.A.I. V M3
14	10	10.6	9.3	7.8	2500	12.0	57					9.3	7.8	2500	12.0	57	57	4.1			
																			5.6	1.9	12.1
16	12	11.8	10.5	8.9	2500	15.7	81	9.8	6.9	1200	23	10.8	10.5	1300	11.2	58	81	5.1			
																			5.9	1.4	10.6
20	16	14.0	12.9	12.8	1300	16.8	100	11.8	9.0	20	0	13.0	12.9	1280	16.7	100	123	6.1			
																			4.9	1.4	12.1
24	20	15.9	14.9	14.8	1280	22.2	148	14.2	12.2	500	38	15.1	16.3	780	16.4	110	171	7.1			
																			4.8	1.0	10.2
29	25	18.0	17.3	18.7	780	21.5	161	15.8	13.1	15	1	17.3	18.8	765	21.3	160	222	7.7			
																			3.8	0.9	10.1
34	30	19.9	19.1	20.7	765	25.8	211	18.5	18.0	300	61	19.4	22.3	465	18.2	150	273	8.0			
																			4.1	0.7	8.1
39	35	21.5	21.1	24.4	465	21.8	190	19.2	17.1	9	2	21.1	24.5	456	21.6	188	313	8.0			
																			3.4	0.6	7.5
44	40	23.0	22.6	26.2	456	24.6	226	20.6	18.4	9	2	22.6	26.4	447	24.4	224	351	8.0			
																			2.8	0.5	6.9
49	45	24.3	23.9	27.8	447	27.1	258	21.8	19.4	8	3	23.9	27.9	439	26.8	255	385	7.9			
																			2.5	0.5	6.5
54	50	25.5	25.1	29.1	439	29.3	288	22.9	20.4	8	3	25.1	29.3	431	29.0	285	418	7.7			
																			2.1	0.4	5.9

TABELL 6 B TREE SIZE DISTRIBUTION Betula Verrucosa per hectare
DIMENSJONSFORDELING BJØRK pr. hektar
H40 = 23

			THINNINGS TYNNING diameter (cm)			AFTER THINNING ETTER TYNNING diameter (cm)		
t1.3	Ho		< 11	>= 11	sum	< 11	>= 11	sum
		Dg	5.7	13.2	6.9	3.8	13.5	10.5
12	11.8	Hl	8.8	11.5	9.8	10.0	11.5	10.8
		N	1075	125	1200	890	410	1300
		V	13.5	9.5	23.0	25.2	32.8	58.0
		Dg		12.2	12.2	11.0	16.9	16.3
20	15.9	Hl		14.2	14.2	13.5	15.2	15.1
		N	0	500	500	112	668	780
		V	0.0	38.0	38.0	7.9	102.1	110.0
		Dg		18.0	18.0		22.3	22.3
30	19.9	Hl		18.5	18.5		19.4	19.4
		N	0	300	300	0	465	465
		V	0.0	61.0	61.0	0.0	150.0	150.0
		Dg					24.5	24.5
35	21.5	Hl					21.1	21.1
		N				0	456	456
		V				0.0	188.0	188.0
		Dg					26.4	26.4
40	23.0	Hl					22.6	22.6
		N				0	447	447
		V				0.0	224.0	224.0
		Dg					29.3	29.3
50	25.5	Hl					25.1	25.1
		N				0	431	431
		V				0.0	285.0	285.0

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