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Diameter distributions and height curves
in even-aged stands of *Pinus Sylvestris* L.

*Diameterfordelinger og høydekurver for
ensaldrede bestand av Pinus sylvestris L.*

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Abstract

MØNNES, E. N. 1982. Diameter distributions and height curves in even-aged stands of *Pinus sylvestris* L. (Diameterfordelinger og høydekurver for ensaldrede bestand av *Pinus sylvestris* L.) Medd. Nor. inst. skogforsk. 36(15): 1—43.

Functions are constructed in order to determine diameter distributions and height curves on stands with known stand parameters. These are the basal area mean diameter, the site index, the tree number per hectare, Loreys height, and the top height. The diameter distribution is based upon Johnsons System b distribution. The distributions are estimated on each stand. The variation of the distributions between stands is regressed against stand parameters.

The type of height curve is a two-parameter hyperbola. The top height and Loreys height, which both are different integrals on the height, together with the diameter distribution determine the height curve completely.

The functions are tested against an independent set of data.

Key words: diameter distribution, Johnsons System b distribution, height curve, restrictions on height curve.

Utdrag

MØNNES, E. N. 1982. Diameter distributions and height curves in even-aged stands of *Pinus sylvestris* L. (Diameterfordelinger og høydekurver for ensaldrede bestand av *Pinus sylvestris* L.) Medd. Nor. inst. skogforsk. 36(15): 1—43.

Det er konstruert funksjoner for å bestemme diameterfordeling og høydekurve for bestand med kjente bestandsdata. Disse er grunnflatemiddeltammens diameter, bonitet, treantall pr. ha, grunnflateveid middelhøyde og overhøyde. Diameterfordelingen er basert på Johnsons System b fordeling. Fordelingen bestemmes på hvert bestand. En modell for fordelingsutviklingen konstrueres ved hjelp av regresjon der variasjon imellom fordelingene forklares med variasjon av bestandsparametrene.

Høydekurvens form er en hyperbel. Overhøyde og grunnflatemiddelveid høyde er begge ulike integraler av høyde. Disse to bestandsparametre vil sammen med diameterfordelingen bestemme høydekurven fullstendig.

Funksjonene er testet mot et uavhengig materiale.

Nøkkelord: diameterfordeling, Johnsons System b fordeling, høydekurve, restriksjoner på høydekurve.

Preface

This work is financed by the Norwegian Agriculture Research Council (NLVF) and the Norwegian Forest Research Institute (NISK). The work was done partly while I was engaged at NISK and partly while I was engaged at the senter for experimental design and data processing, (FDB-sentral). The main computation has been done during the years 1977—1978 using the computer at the FDB-sentral. The material was collected by NISK, division of Forest Management and Yield Studies. Helge Braastad (NISK), Kåre Hobbelstad (NISK), Tore Skrøppa (NISK), and Egil Vestjordet (NISK) have given support at different stages in the work. Lars Strand (NISK) has read the manuscript. Inger Langø (FDB) has typed the manuscript into the computer. The English text was corrected by Sylvia Bredholdt (The Agricultural University of Norway). Ida Svendsen (NISK) has typed formulaes and supported the layout of the computer manuscript. Ole Aavestrud (NISK) has drawn the figures.

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I. Introduction

The aim of this work is to develop functions on diameter distributions and height curves on even aged stands of *Pinus sylvestris*. Data from the stands are partly known. Stand parameters like D_g , H_0 , H_{40} , H_1 , N are to be considered known. (All symbols are defined in appendix 1.) VESTJORDET (1972) has carried out a similar work on *Picea abies*. HAFLEY & SCHREUDER (1976) have examined diameter distributions on single stands. The result of the work is a computer program that with a given set of stand parameters determine the diameter distribution and the height curve. The work was designed to adjust distribution and height curves to «Growth model computer program» (BRAASTAD 1980).

II. Material

The material is used and described earlier by BRANTSEG (1969). By one observation is meant a description of a stand at a given time. One observation may consist of two parts; Thinning and stand after thinning (removed and standing trees). I will frequently talk about an observation meaning only one of these parts.

One observation contains the following information:

$$\begin{aligned} &T_{1.3}, H_0, H_{40} \\ &N_3, H_3, D_3, G_3, V_3, n_3 \\ &N_2, H_2, D_2, G_2, V_2, n_2 \end{aligned}$$

The following values are recorded for both thinning and stand after thinning:

- Diameter distribution, number of trees within 2 cm diameter classes.
- Mean (arithmetic) diameter within 2 cm diameter classes.
- Mean (arithmetic) height of some trees within those diameter classes.

(The volume is calculated due to BRANTSEG 1967. The site index H_{40} is calculated due to TVEITE 1976.) Observations with the following criteria were removed:

- Observations with missing data.
- Observations with less than 10 trees (actual n).
- Observations with less than 5 diameter classes of 2 cm.

589 observations of standing trees and 294 observations of thinned trees then remain. Some further observations were removed due to instability of the estimation process. In order to test the theory developed the material was divided (at random) into two subgroups, an estimation group and a testing group. Table 1 shows the number of observations within each group. Table 2 shows summary statistics of the material.

Table 1. Number of observations within each observation group.

	estimation	testing	sum
thinnings	207	87	294
after thinnings	413	176	589

Table 2. Summary statistics (per hectare).

Variable	N	Mean	Minimum value	Maximum value	Standard deviation
estimation group, thinnings					
T _{1.3}	207	56.4 year	11.0	120.0	26.9
H ₀	207	16.0 m	5.6	27.4	3.6
H ₂	207	13.1 m	4.8	24.3	3.8
H ₄₀	207	14.2 m	7.6	18.9	3.0
D ₂	207	13.4 cm	3.7	27.8	5.0
V ₂	207	21.2 m ³	2.2	67.6	13.4
N ₂	207	439.8	35.0	4803.0	698.8
testing group, thinnings					
T _{1.3}	87	62.3 year	14.0	125.0	26.4
H ₀	87	16.4 m	6.9	26.3	3.6
H ₂	87	13.4 m	6.0	23.7	3.8
H ₄₀	87	13.5 m	7.5	18.9	3.1
D ₂	87	13.4 cm	4.5	26.3	5.3
V ₂	87	21.4 m ³	1.9	80.7	12.4
N ₂	87	438.1	44.0	4068.0	637.1
estimation group, stand after thinning					
T _{1.3}	413	66.1 year	11.0	134.0	28.6
H ₀	413	17.2 m	5.6	27.5	3.7
H ₃	413	15.7 m	5.0	27.1	3.9
H ₄₀	413	13.7 m	6.6	19.1	3.0
D ₃	413	18.2 cm	5.1	41.0	5.9
V ₃	413	185.2 m ³	36.7	468.5	80.1
N ₃	413	1292.0	233.0	7830.0	1215.9
testing group, stand after thinning					
T _{1.3}	176	66.5 year	11.0	137.0	29.7
H ₀	176	16.7 m	5.8	27.6	3.9
H ₃	176	15.2 m	5.4	26.8	4.0
H ₄₀	176	13.4 m	7.6	18.9	2.9
D ₃	176	17.8 cm	6.1	39.9	6.0
V ₃	176	166.9 m ³	28.2	406.4	76.2
N ₃	176	1243,4	233.0	7350,0	1113,0

III. Methods

A diameter distribution is a function that to every real number assigns the (relative) number of trees with a diameter less than or equal to the given real number. A height curve is a function that to every diameter assigns a height. The function will vary with site, age and management. Certainly the number of trees with diameter less than, say 10 cm, will decrease both by age and thinnings. A material with completely known stands is given. Functions must be established that, to a given set of stand parameters, determine the distribution and the height curve. One way of doing this is to select a specific class of distributions described by some parameters. These parameters should be estimated on each stand. The variation from stand to stand is supposed to depend on stand parameters. A multiple regression model is used. The height curve may be handled in a similar fashion. It is not obvious that regression should be used. Stand parameters are not only «independent» variables, they are properties of the distribution and height curve themselves. Thus the distribution should not only be explained by the D_g (e.g.) but it should also achieve the D_g on each stand.

A. Diameter distribution on each observation

We have to choose a theoretical distribution function to fit the observed diameter distribution. Attention should be drawn to the following arguments:

The distribution should conform with observed distributions.

Estimation of the parameters.

Calculation of other properties.

The Johnson System b distribution (abbr. J-Sb) (JOHNSON 1949; JOHNSON & KOTZ 1970) has been chosen to fit the diameter distribution. The distribution has been used to fit diameters by HAFLEY & SCHREUDER (1976). The bivariate J-Sb (JOHNSON & KOTZ 1972) has been used on diameter/height distributions by SCHREUDER & HAFLAY (1977).

The J-Sb density is

$$f(d; \xi, \lambda, \delta, \gamma) =$$

$$\frac{\delta}{\sqrt{2\pi}} \frac{\lambda}{(d-\xi)(\lambda+\xi-d)} \exp^{-\frac{1}{2} \left[\gamma + \delta \ln \left[\frac{d-\xi}{\lambda+\xi-d} \right] \right]^2} \quad (\text{III.A.1})$$

$$\xi \leq d \leq \xi + \lambda, \quad -\infty < \xi < +\infty, \quad \lambda > 0, \quad -\infty < \gamma < +\infty, \quad \delta > 0.$$

The parameters ξ, λ are location and scale parameters. δ, γ determine the «form» of the distribution. Growing δ implies more excess. Growing absolute value of γ implies more skewness.

if D is distributed as a J-Sb then

$$Z = \gamma + \delta \ln \left[\frac{D - \xi}{\lambda + \xi - D} \right] \quad (\text{III.A.2})$$

is distributed as a standard normal variable.

The cumulative distribution

Probability of a random diameter to be less than or equal to the number $d = P(D \leq d) =$

$$\begin{aligned} & \int_{-\infty}^d f(x; \xi, \lambda, \delta, \gamma) dx = \\ & \int_{\xi}^d \frac{\delta}{\sqrt{2\pi}} \frac{\lambda}{(x-\xi)(\lambda+\xi-x)} \exp^{-1/2 \left[\gamma + \delta \ln \left[\frac{x-\xi}{\lambda+\xi-x} \right] \right]^2} dx = \\ & \Phi \left[\gamma + \delta \ln \frac{d-\xi}{\lambda+\xi-d} \right] \end{aligned} \quad (\text{III.A.3})$$

Φ is the standard cumulative normal distribution. All moments exist but the formulae are complicated (JOHNSON 1949).

Fractiles

A fractile is a real number (diameter) such that a certain amount of trees have a diameter less than or equal to that number.

Compute d_α such that

$$P(D \leq d_\alpha) = \alpha \quad 0 < \alpha < 1 \quad (\text{III.A.4})$$

(III.A.4) together with (III.A.3) gives

$$d_\alpha = \xi + \lambda \left[1 + \exp \left[\frac{\gamma - \Phi^{-1}(\alpha)}{\delta} \right] \right]^{-1} \quad (\text{III.A.5})$$

where Φ^{-1} is the inverse Φ ; $\Phi(\Phi^{-1}(\alpha)) = \alpha$.

The median is

$$d_{0.5} = \xi + \lambda \left[1 + \exp \left[\frac{\gamma}{\delta} \right] \right]^{-1} \quad (\text{III.A.6})$$

A distribution «*shape*» may be considered given by its 3rd and 4th moment, or, equivalently, by its place in the $\beta_1 \cdot \beta_2$ space (skewness · kurtosis).

The J-Sb is one of three transformations of a normal distributed variate proposed by JOHNSON (1949) which together cover the skewness · kurtosis space. The J-Sb covers all possible space «above» the lognormal curve. Some examples of J-Sb are given in Fig. 1.

The (β_1, β_2) parameters are estimated on the observed diameter distributions and plotted in the $\beta_1 \cdot \beta_2$ space in Figs. 2, 3, 4, 5. (The plots shows the observed skewness and kurtosis of the stands. The upper border is the limit of the possible $\beta_1 \cdot \beta_2$ space. The lower border is the possible combinations of the lognormal distribution. The space in between is the possible area of J-Sb. The point (0,3) is achieved by the normal distribution.)

The diameter distributions typically place themselves in the J-Sb region. HAFLEY & SCHREUDER (1976) also obtained this result.

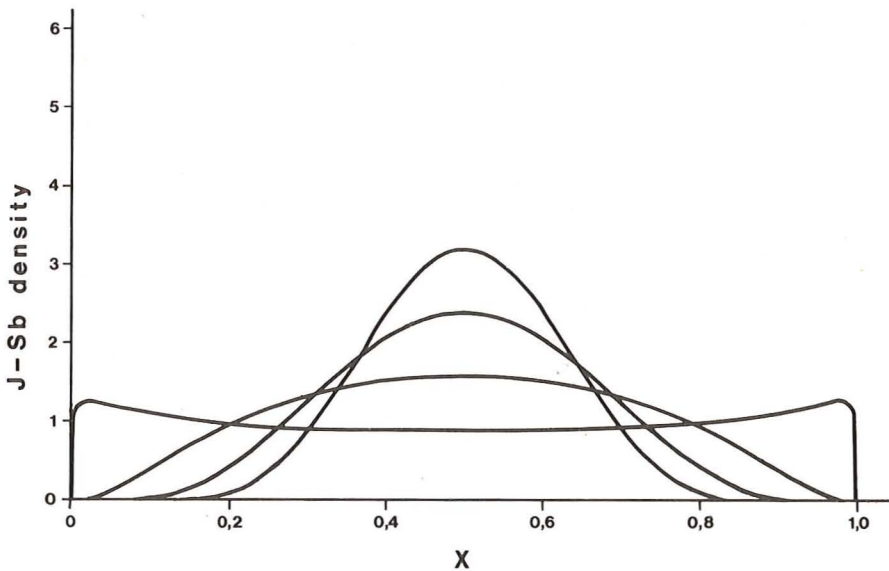


Fig. 1a. Some symmetric J-Sb distributions.

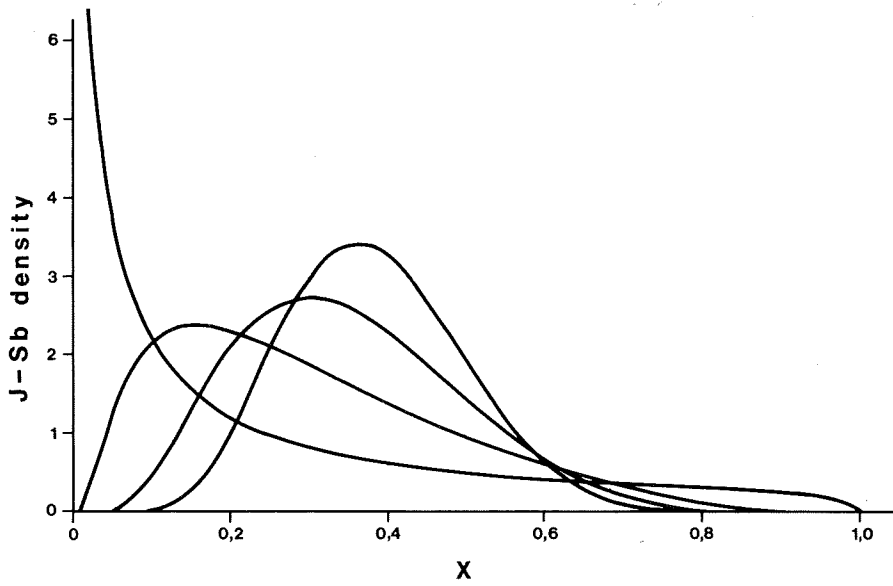


Fig. 1b. Some right-skewed J-Sb distributions.

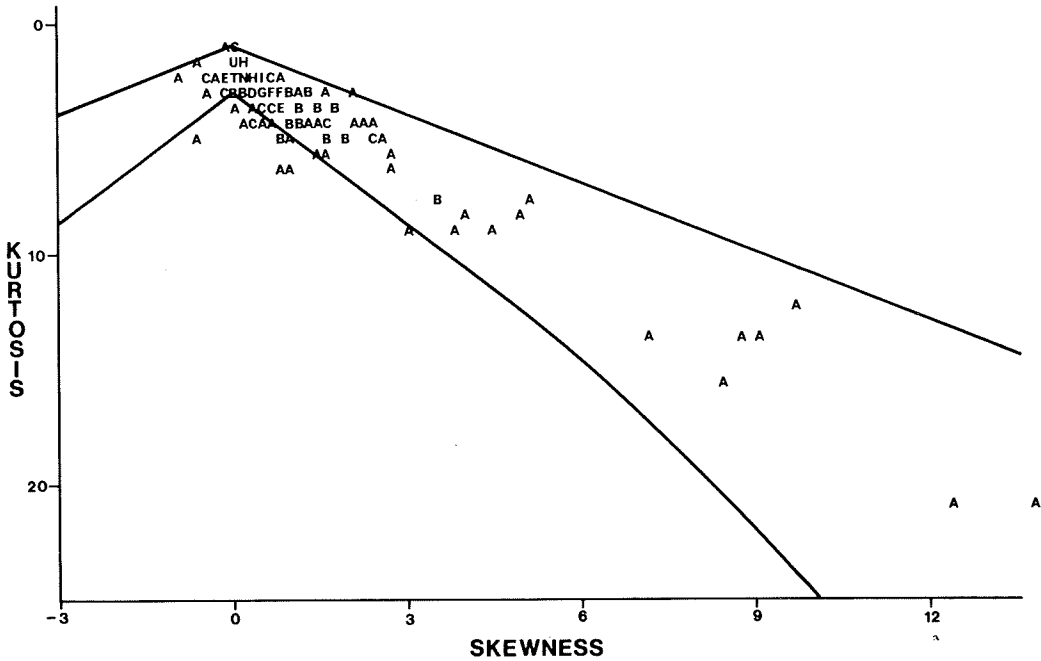


Fig. 2. Thinnings, estimation group. A=1 obs, B=2 obs, etc. The upper border is the limit of $\beta_1 \times \beta_2$ space. The lower border is the lognormal curve. The space inbetween is the J-Sb area.

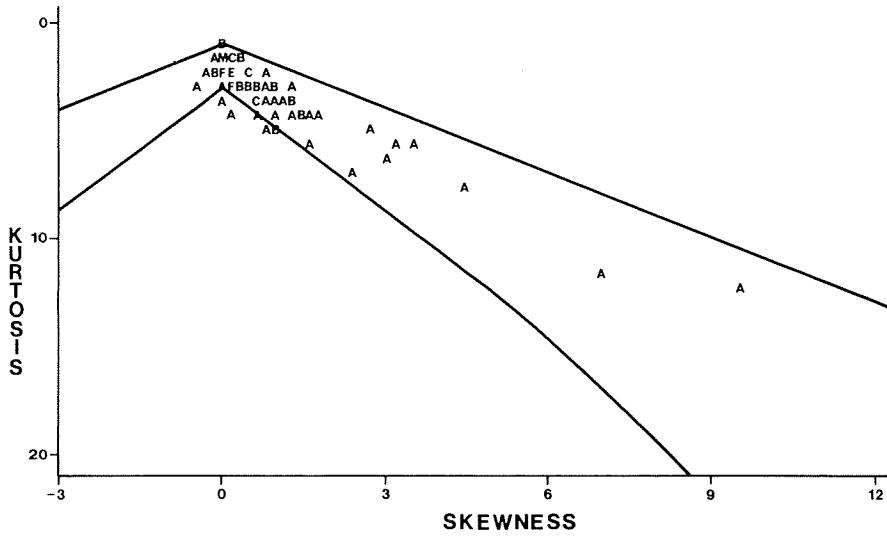


Fig. 3. Thinnings, testing group. A=1 obs, B=2 obs, etc. The upper border is the limit of $\beta_1 \times \beta_2$ space. The lower border is the lognormal curve. The space inbetween is the J-Sb area.

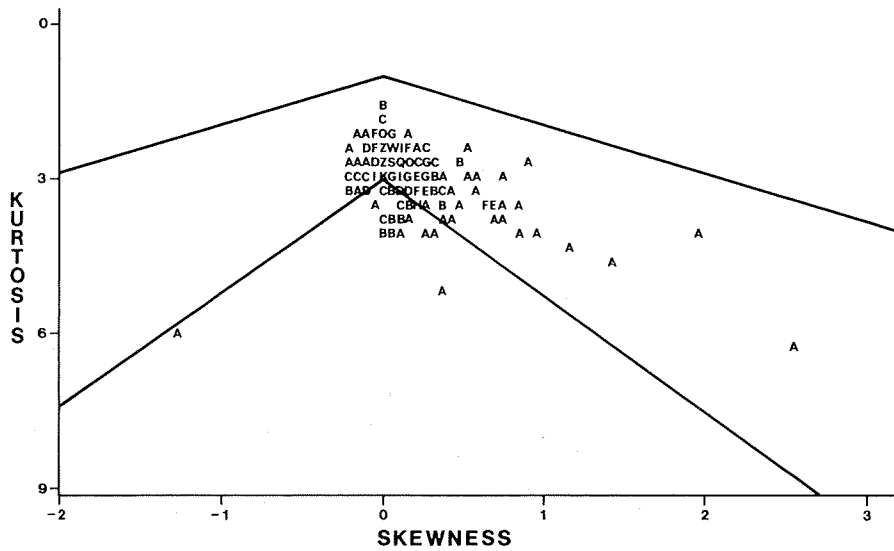


Fig. 4. Standing trees, estimation group. A=1 obs, B=2 obs, etc. The upper border is the limit of $\beta_1 \times \beta_2$ space. The lower border is the lognormal curve. The space inbetween is the J-Sb area.

$$\begin{aligned}
 \xi(T) &= M_0' \cdot T \\
 \lambda(T) &= M_1' \cdot T \\
 \delta(T) &= M_2' \cdot T \\
 \gamma(T) &= M_3' \cdot T
 \end{aligned}
 \tag{III.B.1}$$

gave a very poor fit except for $\xi(T) = M_0' \cdot T$.

(Different stand parameters may «explain» different distribution parameters. (III.B.1) do handle this, elements of the M's may of cause be zero.)

This result could have been foreseen. It is difficult to imagine how the J-Sb parameters should vary with stand parameters. The following transformation of the parameters was then considered. We use (III.A.5) to compute 4 fractiles and try regression on them instead. It is more likely that fractiles should fit into a regression model with stand parameters. Fractiles have a direct physical interpretation in the stand. The median and the D_g are of same kind of data and they will a priori be highly correlated. We choose (to ease calculations) the four fractiles $\alpha=0$, $\alpha=0.3085$, $\alpha=0.5$, $\alpha=0.6914$ (inverse normals are then $-\infty$, -0.5 , 0.0 , 0.5 , respectively).

We have

$$d_0 = \xi \tag{D_{min}}$$

$$d_{0.31} = \xi + \lambda \left[1 + \exp \left[\frac{\gamma + 0.5}{\delta} \right] \right]^{-1}$$

$$d_{0.5} = \xi + \lambda \left[1 + \exp \left[\frac{\gamma}{\delta} \right] \right]^{-1} \tag{the median}$$

$$d_{0.69} = \xi + \lambda \left[1 + \exp \left[\frac{\gamma - 0.5}{\delta} \right] \right]^{-1} \tag{III.B.2}$$

We now fit the model

$$d_i(T) = M_i' \cdot T \quad i = 0, 0.31, 0.5, 0.69 \tag{III.B.3}$$

The M vectors are estimated independent of each other. This model has an acceptable fit. (III.B.3) will be used. The M's and their goodness of fit are given in table 3.

For a given T (III.B.3) gives rise to 4 real numbers. For these numbers the equations (III.B.2) must be inverted to obtain the J-Sb parameters. This is not always possible. Any four numbers may not be considered as four fractiles of some J-Sb.

We have the following lemma:

A necessary and sufficient condition on four real numbers (z_0, z_1, z_2, z_3 , e.g.) to be considered as four fractiles

($d_0, d_{0.5-p}, d_{0.5}, d_{0.5+p}$) ($0 < p < 0.5$) of some J-Sb

is that

$$z_0 < z_1 < z_2 < z_3 \quad (\text{III.B.4})$$

and

$$(z_2 - z_0)^2 - (z_3 - z_0)(z_1 - z_0) > 0 \quad (\text{III.B.5})$$

The J-Sb corresponding to these four numbers when $p=0.19$ (our choice) is given by

$$\xi = z_0$$

$$\lambda = \frac{(z_2 - z_0)((z_2 - z_0)(z_1 - z_0) + (z_2 - z_0)(z_3 - z_0) - 2(z_1 - z_0)(z_3 - z_0))}{(z_2 - z_0)^2 - (z_1 - z_0)(z_3 - z_0)}$$

$$\delta = -\frac{1}{2} \left[\ln \frac{(z_3 - z_0)(\lambda + \xi - z_2)}{(\lambda + \xi - z_3)(z_2 - \xi)} \right]^{-1}$$

$$\gamma = \frac{1}{2} - \delta \ln \left[\frac{z_3 - z_0}{\lambda + \xi - z_3} \right] \quad (\text{III.B.6})$$

Proof: Verified by direct calculations.

Fractiles of the form (III.B.2) fulfill (III.B.4) and (III.B.5). On the other hand λ of (III.B.6) has to be greater than $z_3 - z_0$. This will only happen when (III.B.5) holds.

(The choice of $p=0.19$ is in (III.B.6) seen from the number 0.5 in the formulae of δ any γ). (III.B.3) gives raise to a J-Sb only when (III.B.5) is fulfilled. This defines a subspace of T which is acceptable. In this material some observations did not give estimates fulfilling (III.B.5). For these a J-Sb may not be found with the functions. This was typical of the cases where tree density and D_g were simultaneously high. (III.B.3) imposes restrictions on the J-Sb found. The freedom of J-Sb to fit different forms will be limited. This depends on the complexity of T. If T is one-dimensional (e.g. T = age) the J-Sb's raising from (III.B.3) will differ only in scale and location. However, as soon as T becomes two-dimensional, we obtain some freedom of form. The distribution of thinnings will be seen to depend only on D_g and a constant term. The distribution changes from righthand to lefthand skewness as D_g grows.

D_g is used as an independent variable to predict a distribution. This distribution of cause has a « D_g ». The two D_g 's should be equal. Ideally this should be a restriction on the regression. This has not been done here. The discrepancy will be shown to be negligible.

C. Height

In this work J-Sb is used on diameters. The outlined theory on height does not depend on this, it will succeed on all types of diameter distributions. The actual formulae will, in any case, depend on J-Sb.

The data presented may not be used to establish a height distribution. We must restrict ourselves in finding a function that determine a height for each diameter. VESTJORDET (1972) used the function

$$h = \left[\frac{d}{A+Bd} \right]^3 + 1.3 = \left[A\frac{1}{d} + B \right]^{-3} + 1.3 \quad (\text{III.C.1})$$

in his work on Norway spruce. He also considered it as «best» among some others. On each observation we have tried this and also

$$h = \left[\frac{d}{A+Bd} \right]^2 + 1.3 = \left[A\frac{1}{d} + B \right]^{-2} + 1.3 \quad (\text{III.C.2})$$

and that implicitly given by

$$\ln \left[\frac{h-h_{\min}}{h_{\max}-h} \right] = \ln \left[\frac{d-\xi}{\xi+\lambda-d} \right] A + B \quad (\text{III.C.3})$$

The last curve is the median regression of the bivariate J-Sb distribution. Even without a bivariate distribution its conditional properties may be used. The main difficulty will be to estimate h_{\min} and h_{\max} .

(III.C.1) has a transformation to a linear form:

$$(h-1.3)^{-\frac{1}{3}} = A\frac{1}{d} + B \quad (\text{III.C.4})$$

A and B were estimated on each observation and the fit was measured. The transformation may bias the estimation. As the estimates will not be used later, we will abandon that discussion here.

We did not discover any interesting differences in the three curves. We decided to use (III.C.1).

Some stand parameters are related to the height curve. In this data H_1 and H_0 are given. H_1 and H_0 are of cause dependent of the height curve. On the reverse, we will now prove that the height curve (III.C.1) is completely determined by H_1 , H_0 and the diameter distribution.

H_1 is the weighted mean height, the area (squared diameter) being the weights. H_0 is the mean height among the 100 thickest trees (pr. hectare). That is the mean height among trees thicker than the $(1 - 100/N)$ fractile diameter. This may be approximated by the following:

H_0 is the height of the $(1 - 50/N)$ fractile diameter.
That is, H_0 is the height of the diameter (D_0 , e.g.).

$$D_0 = d_{1-\frac{50}{N}} = \xi + \lambda \left[\gamma + \delta \ln \frac{\gamma - \phi^{-1} \left[1 - \frac{50}{N} \right]}{\delta} \right]^{-1} \quad (\text{III.C.6})$$

Note that D_0 only depends on the diameter distribution.

Inserting (III.C.6) in (III.C.1) gives

$$H_0 = h(D_0) = \left(A \frac{1}{D_0} + B \right)^{-3} + 1.3$$

Which is equivalent to

$$B = (H_0 - 1.3)^{-\frac{1}{3}} - \frac{A}{D_0}$$

This B reinserted in (III.C.1) gives

$$h(d) = \left(A \left(\frac{1}{d} - \frac{1}{D_0} \right) + (H_0 - 1.3)^{-\frac{1}{3}} \right)^{-3} + 1.3 \quad (\text{III.C.7})$$

Now consider the H_1 measure and substitute (III.C.7) for height:

$$H_1 = \frac{\sum (h \frac{g}{\sum g})}{\sum g} =$$

$$\frac{\sum d^2 \left(\left(A \left(\frac{1}{d} - \frac{1}{D_0} \right) + (H_0 - 1.3)^{-\frac{1}{3}} \right)^{-3} + 1.3 \right)}{\sum d^2} \quad (\text{III.C.8})$$

(III.C.8) is an equation in the single parameter A which is easily solved by Newton's method. In this context A is unique because (III.C.1) is monotone in A. The procedure will not converge if $H_1 = H_0$. ($A=0$ is then a solution, giving a constant height to every diameter.) (When using theoretical distributions, (III.C.8) should really be an integral. We use 1 cm diameter classes in our algorithm.)

The outlined theory is used as follows:

For a given set of stand parameters, we use (III.B.3) and (III.B.6) to determine its diameter distribution. Together with H_1 and H_0 this diameter distribution determine the height curve (III.C.7) by solving (III.C.8). There is one problem left with the thinnings: Usually no « H_0 trees» are thinned. These trees are predominant in the stand after thinning. To fix the height curve at H_0 we have to use D_0 from the stand after thinning. Thus both the distribution of the thinning and that after thinning has to be known.

It is to be emphasised that the height curve does not depend on any kind of estimation involving heights. The curve is a mathematical consequence of (III.C.1), H_1 , H_0 and the estimated diameter distribution.

D. Controlling results

The calculations evolve through different stages. Selection of a distribution function (and a height curve), estimation of distribution parameters, regression of distribution on stand parameters. At each stage of calculation a discrepancy between real world and model is introduced. Tests at each stage are dependent upon the previous stage. With these tests we have no control on the overall significance. On the test group, however, we may perform overall tests of significance.

I do not wish any of the hypothesis tested to be rejected. On the other hand discrepancy between model and real world will always be the case. If the tests are powerful enough, this discrepancy will be detected by the tests. One should not therefore rely on these tests alone. Some judgement should be made from the forestry point of view. One has to ask if the model meets the needs of a forester. All observations have been plotted together with the estimated distributions to visualise the fit.

Testing on diameters

Diameters are supposed to be realisations of some kind of continuous distribution. Diameters are not observed directly here due to grouping. They constitute a multinomial distribution, generated from a continuous distribution. (We also know the arithmetic mean within each group.) Introducing J-Sb to each observation may be tested by

«The observation is generated from some J-Sb»
against
«The observation is arbitrary» (III.D.1)

«Arbitrary» means that we make no assumptions on the underlying distribution. The hypothesis is that some J-Sb, with some unknown parameters, is an adequate description of the observed distribution. Using the multinomial distribution, (III.D.1) may be tested by both the Karl-Pearson test and the likelihood test. In both tests we have used the estimates from chapter III.A. In the regression (III.B.3) stepwise regression have been used to select independent variables. (III.B.3) yields a J-Sb to each observation, based on its stand parameters. The test may be performed against different alternatives:

«The observation is generated from the J-Sb determined by the functions (III.B.3)»
 against (III.D.2)
 «The observation is generated from some (arbitrary) J-Sb»
 or
 «The observation is generated from the J-Sb determined by (III.B.3)»
 against (III.D.3)
 «The observation is arbitrary»

(III.D.2) may be tested using the likelihoods from the continuous distributions. (III.D.3) and (III.D.1) can only be tested using the multinomial distributions. (III.D.3) is the test which is of interest to a forester. The diameter functions are tested against real observations. To a statistician it may be of interest to divide the test into two parts: one concerning the use of J-Sb and the other concerning the further use of the diameter functions (III.B.3). Using likelihood (discrete) test, the test from (III.D.1) and (III.D.2) are asymptotically independent and their Chi-square sums add up to the Chi-square sum of (III.D.3). (This is only true if the number of diameter classes are unaltered under the estimation of the functions (III.B.3).)

The actual number of trees (n_2, n_3) on each stand is varying, some times being rather high. The test power is therefore high. Small discrepancies will be detected and give high Chi-square sums. Also, the estimated interval of diameters (from (III.B.3)) may not coincide with the actual interval of observed trees. This gives a high Chi-square sum even if the major part of the distribution fits well. One way of bypassing this is to omit the Chi-square test on each stand. Instead, we add the frequencies within each 2 cm class, giving the distribution throughout the whole «forest». The three tests (III.D.1), (III.D.2), (III.D.3) may be performed on these multinomial distributions. Histograms concerning these tests are given in figs. 6, 7, 8, 9.

Testing of heights

The type of height curve is selected by calculating the regressions (III.C.4), (III.C.2) (transformed) and (III.C.3) on each observation. As the further work on heights does not involve any estimation, there is nothing more to test. However, we may test the theoretical heights (h) against the observed heights (H) on the testing material. As we should have

$$H \approx h$$

over all diameter classes and all observations, we consider the model

$$H = c + b \cdot h + \text{error.}$$

(The error structure is taken to be independent and identically normal.)
 The hypothesis of interest is

$$c=0, b=1 \text{ against } c, b \text{ arbitrary.} \quad (\text{III.D.4})$$

We have also compared the heights within each diameter class (Fig. 10 and 11).

Other tests

The distribution raised from (III.B.3) yields a theoretical D_g . This distribution together with the theoretical height curve yields a theoretical volume. The (III.D.4) kind of test may be applied on D_g and volume as well.

IV. Results

The result of this work is a computer program that for each set of stand parameters determines the diameter distribution and the height curve. In the program the volume is adjusted so that it will always equal the given volume. The output of the program is designed to fit NISK's usual distribution tables.

The text below is concerned with the validity of the functions. The regression coefficients of (III.B.3) are given in Table 3. These are the functions to be used in determining the distribution of some stand. The fit in the central part of the distribution is very satisfactory. (11 thinned observations have been removed from the regression. The estimation process (of appendix 2) did not converge properly.)

Table 3. Regression coefficients, diameter distribution.

Variable	thinnings				stand after thinning			
	d_0	$d_{0.3}$	$d_{0.5}$	$d_{0.69}$	d_0	$d_{0.3}$	$d_{0.5}$	$d_{0.69}$
D_g	0.543	0.927	1.013	1.088	1.470	1.278	1.168	1.074
H_{40}	0	0	0	0	0.158	0.079	0.041	0
$D_g \cdot \ln(N)$	0	0	0	0	-0.174	-0.066	-0.030	0
constant	-2.143	-1.357	-0.888	-0.105	-0.592	-0.997	-0.675	0.168
R-square	0.58	0.97	0.99	0.99	0.62	0.97	0.99	0.99
obs	198				413			

Selection of height curve was based on Table 4. None of the curves was exceptional. The height curve (III.C.1) was used.

Table 4. R-square, height curve on each stand (linear form).

	formulae	obs	mean	R-squares		
				max	min	sd
after thinning	(III.C.1)	413	0.93	1.00	0.35	0.08
	(III.C.2)	413	0.93	1.00	0.35	0.08
	(III.C.3)	413	0.90	1.00	0.32	0.09
thinnings	(III.C.1)	209	0.92	1.00	0.23	0.10
	(III.C.2)	209	0.92	1.00	0.23	0.10

All diameter tests, performed on each observation, are given in Table 5. As all of them end up with Chi-square, only the Chi-square sums are given. (A sum of independent Chi-squares is also a Chi-square.)

(III.D.3) test the functions against observed stands.

(III.D.1) test use of J-Sb against observed stands.

(III.D.2) test the functions against use of J-Sb.

In Table 5, «¹⁾» denotes that cells have been added together to obtain cell expectations greater than 5. This must be done to ensure Chi-square approximation. This reduction of cell number will sometimes give zero degrees of freedom. These observations are removed from the test. (The uncorrected Karl-Pearson test may have a poor Chi-square approximation.) The likelihood test (discrete) is based upon the multinomial distribution (as the Karl-Pearson test). The likelihood test (continuous) is based upon the likelihoods from the J-Sb distribution. The « $-2 \ln$ » rule is used to achieve Chi-square statistics in the likelihood tests.

Table 5. Results from individual tests on diameter distribution on each observation.

	no. obs	no. trees	df	Chi-square
estimation group, thinnings				
(III.D.1) Karl-Pearson test ¹⁾	31	4558	50	168
» »	207	13827	647	3730
» , likelihood (discrete) test	»	»	»	1623
estimation group, stand after thinning				
(III.D.1) Karl-Pearson test ¹⁾	394	78646	1264	3040
» »	413	84600	2427	8403
» , likelihood (discrete) test	»	»	»	5437
testing group, thinnings				
(III.D.3) Karl-Pearson test ¹⁾	82	6221	250	972
» »	86	6264	604	6779
» , likelihood (discrete) test	»	»	»	2132
(III.D.1) »	79	5851	225	589
(III.D.2) »	»	»	4.79	1041
» , likelihood (continuous) test	75	5735	4.75	1815
testing group, stand after thinning				
(III.D.3) Karl-Pearson test ¹⁾	175	34860	1192	4705
» »	»	»	1646	19820
» , likelihood (discrete) test	»	»	»	8186
(III.D.1) »	137	27350	676	1384
(III.D.2) »	»	»	4.137	2440
» , likelihood (continuous) test	127	25058	4.127	2726

¹⁾ denotes that cells have been added to give expectation of cell ≥ 5 .

Summary Chi-square tests, where each 2 cm class frequency has been added up over all observations to give the distribution in the whole «forest» are given in Table 6. (In this test observations with $10000/(H_0 \cdot \sqrt{N}) < 12$ has been removed.) Histograms of the «distributions» are given in Figs. 6, 7, 8, 9.

Table 6. Summary Chi-square test.

	no. trees	no classes	df	Chi-square
testing group, thinnings (III.D.1)	5876	17	?	14.68
(III.D.3) (overall test)	»	»	16	82.21
testing group, after thinning (III.D.1)	32942	26	?	94.86
(III.D.3) (overall test)	»	»	25	119.71
estimation group, thinnings (III.D.1)	13347	18	?	478.47
estimation group, after thinning (III.D.1)	73648	26	?	76.38

«?» in place of df indicate that this is not really a Karl Person test

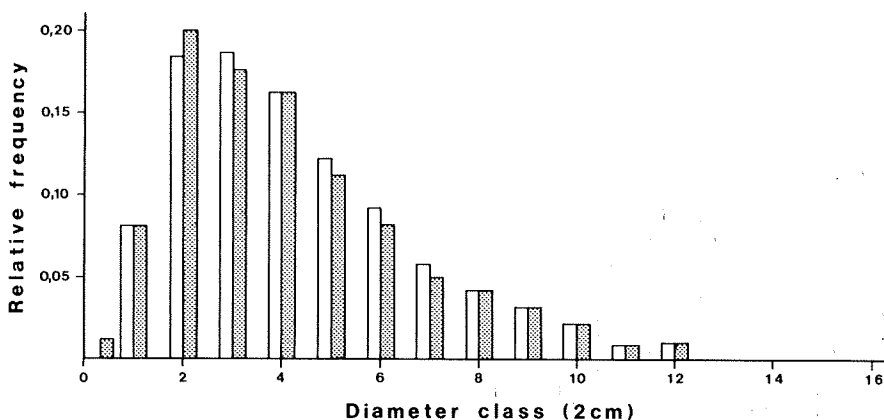


Fig. 6. Thinnings, testing group. Distribution of all trees within each diameter class. White column: predicted by the functions. Shaded column: observed.

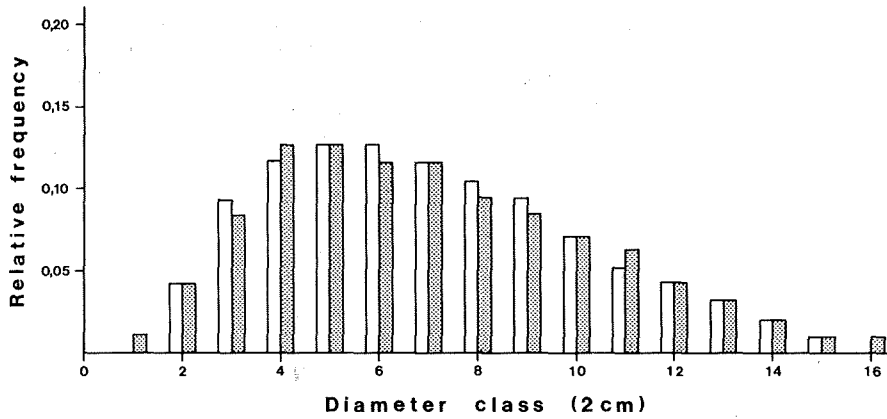


Fig. 7. Standing, testing group. Distribution of all trees within each diameter class. White column: predicted by the functions. Shaded column: observed.

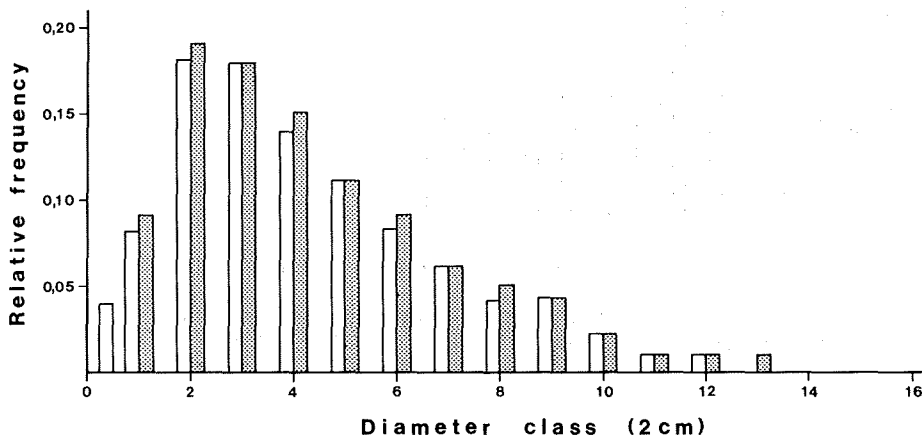


Fig. 8. Thinnings, estimation group. Distribution of all trees within each diameter class. White column: Predicted by J-Sb on each observation. Shaded column: observed.

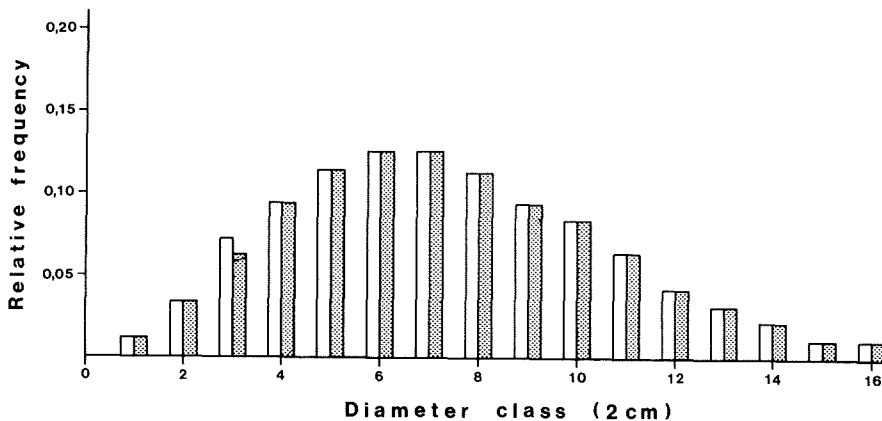


Fig. 9. Standing, estimating group. Distribution of all trees within each diameter class. White column: Predicted by J-Sb on each observation. Shaded column: observed.

Summary statistics on height, D_g and volume are given in Table 7. The height given by the height curve is compared with the observed height within each diameter class in every observation. The D_g and the volume given by the diameter function and height curve is compared with the observed on each observation. The hypothesis tested is (III.D.4), equality against linearity. The linearity is highly significant.

Table 7. Height, D_g and volume (testing group) (calculated parameters with lower case signs).

	estimated function	sigma	df	F(2,df)	R ²
thinnings	$D_2 = -0.11 + 1.01 \cdot d_2$	0.05	84	67.8	.99
	$V_2 = 0.54 + 0.94 \cdot v_2$	1.77	84	13.9	.98
	$H_2 = 0.17 + 0.98 \cdot h_2$	0.92	633	6.0	.96
after thinning	$D_3 = 0.02 + 1.001 \cdot d_3$	0.18	173	3.5	.99
	$V_3 = -6.76 + 1.037 \cdot v_3$	8.57	173	9.0	.99
	$H_3 = 0.20 + 0.98 \cdot h_3$	0.84	1849	44.9	.97

The mean residual heights are plotted against diameter class in Figs. 10 and 11. Plots of some single stands, including observed and predicted diameter distributions and height curves, are given in appendix 3.

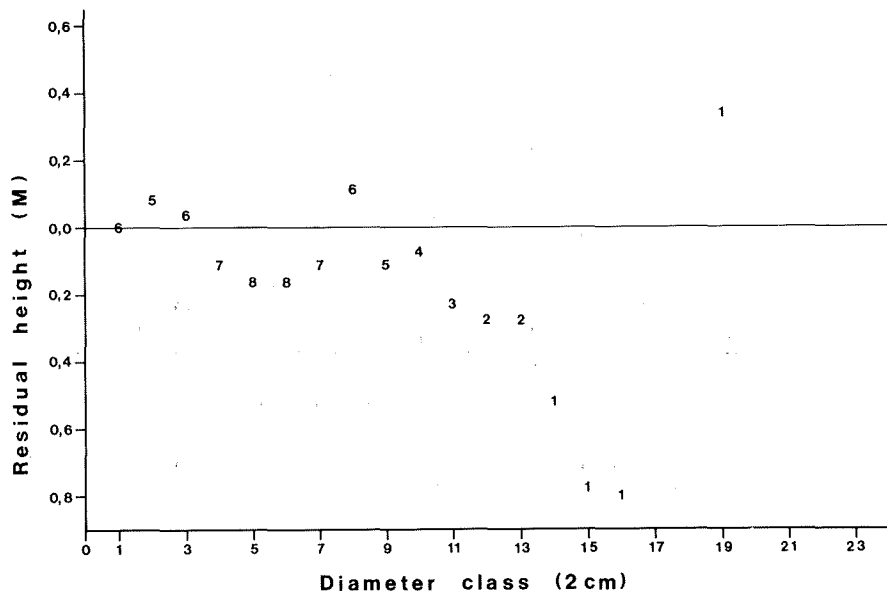


Fig. 10. Thinnings, testing group. Residual mean height. 1=1—9 measures, 2=10—19 measures, etc.

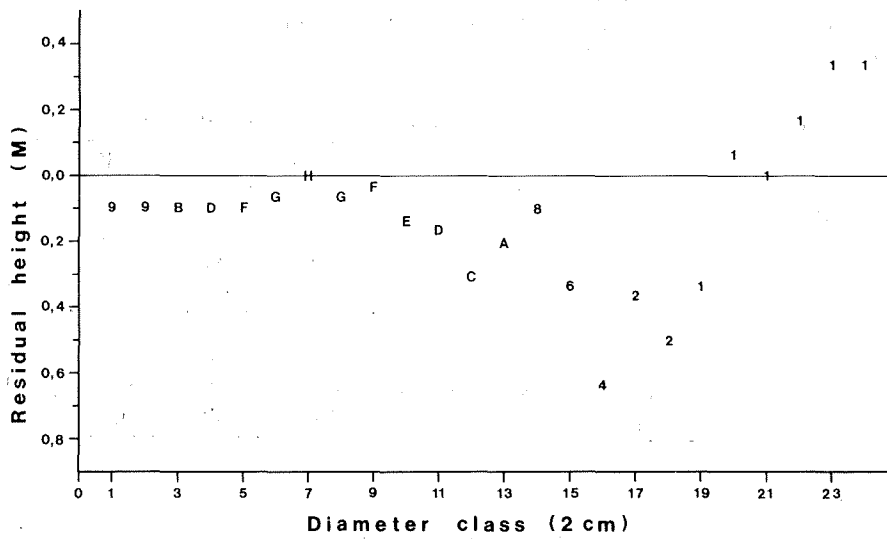


Fig. 11. Standing, testing group. Residual mean height. 1=1—9 measures, 2=10—19 measures,, A=90—99, B=100—109, etc.

V. Discussion

The tests of chapter IV. give poor support to my functions. We shall study Table 5 in some detail, taking the figures on testing group, stand after thinning. It should be noted that a reduction of stand area and thereby the number of trees (without altering the distribution) would reduce the Chi-square sum accordingly.

The likelihood test (III.D.3) rejects the functions. To study the origin of this deviance we must study the observations where the predicted number of diameter classes equals the observed. This is the case in 137 observations. The test (III.D.1) tests the use of J-Sb. (III.D.2) (with $4 \cdot 137$ df) tests the difference between the functions and the use of J-Sb. The overall test (that is (III.D.3)) on these 137 observations is the sum of these tests. From this we conclude that the functions give rise to greater deviance than J-Sb. Also the difference between this sum and the test (III.D.3) of Table 5 tests the 38 observations in between. The Chi-square sum of them is very high. Common to these observations is that the locations of observed and predicted diameters is displaced.

The Figs. 6, 7, 8, 9 give the sum distribution of the whole material. Even if Chi-square tests rejects the functions they may be acceptable to a forester. On stands after thinning there are only 5 classes with a discrepancy of more than 100 (0.3 %) trees between calculated and observed distribution. The classes 7.5—9.5 cm and 9.5—11.5 cm have altogether 310 trees less than observed, while the classes 11.5—13.5, 15.5—17.5 and 17.5—19.5 cm have altogether 474 trees too many. (All from a total of 32 942 trees.)

Table 7 rejects equality between observed and calculated height, volume and D_g . The discrepancy is not very high. The calculated height curves seem to overestimate height when height is higher than about 10 m. ($0.2 + 0.98 \cdot h$ is smaller than h if h is larger than 10 m.) It should be noted that observed parameters are calculated from 2 cm diameter classes while the calculated are based on 1 cm classes.

In Fig. 10 and Fig. 11 the mean differences between observed and calculated heights are plotted against diameter class. The heights are too high for the higher diameter classes. The calculated heights are restricted to meet the given H_0 and H_1 . As these measures depend on the diameter distribution different observations may have equal H_0 and H_1 even if the height curve differ. The deviance here may be explained by an underestimation of the diameter of the H_0 tree (D_0). This may be caused either by the fractile approximation to D_0 or by a tail of the estimated diameter distribution that is too light. The functions to determine the distribution are based upon D_{\min} and 3 central fractiles. The right tail has then to adjust itself. A poor right tail may be the result. If larger diameters are more important than smaller D_{\max} should substitute D_{\min} in the calculations (III.B.3).

Plots of single stands have been studied. Some of these are given in appendix 3. From these we may also see how the form of the distribution changes with stand parameters.

The diameter distribution should show consistency when a stand is watched during its lifetime. Typically a tree should always be growing. A thin-

ned tree should exist in the foregoing standing distribution. There should therefore be a consistency between distribution of thinnings and stand after thinning. Functions of thinnings and stand after thinning have been developed independent of each other. No restrictions were laid down in order to assure consistency. BRAASTAD (1980) used the same functions during observation of stand during several thinnings. No inconsistency between thinnings and stand after thinnings are shown there. Thus, with consistent development of stand parameters, the distribution calculated shows consistency. The same kind of arguments may be used on the height curve. No height should ever decrease. As this curve is a mathematical construction from given H_1 , H_0 and a calculated distribution, consistency in diameter distribution assures a consistency in the height curve. No attempt was made to restrict the distribution so that it will always achieve the given D_g . Table 7 shows that the discrepancy is small. This is the case because there is observed a very strong linear relation between the percentiles of the J-Sb distribution and D_g (Table 3). if the D_g of the J-Sb had been easier to calculate from parameters, we should have used that relation to restrict the distribution in order to achieve the given D_g . In the normal distribution (when it is not truncated) there is an easy relation between D_g and distribution parameters and the restriction is easily laid. This is not easily transferred to J-Sb fractiles because the probability integral up to D_g in normal distribution is not location invariant.

If I was previously aware that fractiles of J-Sb were to be used to develop the diameter functions this would have been a strong argument favouring an easier estimation method of J-Sb on each stand. In chapter III.A. we outlined three different estimation methods. If we used the fractile method these fractiles could be used in the regression (III.B.3) thus we would save a lot of computation. Anyway the method used is «best» in the sense that it theoretically gives estimates with smaller variances.

Many distribution functions have been used on diameters during time. A work that should also be done in Norway is to compare them. Also if a height measure on each tree was available it would be of interest to analyse the multidimensional distribution of diameter and height.

Summary

The aim of this work is to construct mathematical functions that describe the diameter distribution and height curve of stands of even aged *Pinus sylvestris*. Stand parameters are considered known. Thus the functions are to give distribution and height curve with known properties like D_g , H_1 , H_0 on each stand. A material with completely known stands is given. The material is divided into two groups called estimation and testing group, respectively.

The Johnson System b (J-Sb) distribution is used to fit distributions. J-Sb ((III.A.1), (III.A.3)) is a four parameter distribution which changes in location, scale and «form». The region of skewness and curtosis that J-Sb covers is seen to coincide with the region in which the diameter distributions are mainly located. (Figs. 2, 3, 4, 5). The J-Sb parameters have been estimated by the maximum likelihood method on each stand.

The variation in distribution from observation (stand) to observation is handled with a multiple regression model. The J-Sb parameter is not used directly; we transform them into four fractiles (III.B.2) which are regressed (III.B.3). No effort is made to restrict the distribution to achieve the given D_g on each stand. The D_g found is nearly equal to the given (Table 7). The regression functions are given in Table 3. The J-Sb parameters are then found from the transformation (III.B.6). Calculation of the distribution is done by (III.A.3).

The curve (III.C.1) is used to fit heights. As H_0 and H_1 are given the curve should reach these measures on each stand. A mathematical algorithm is developed to ensure this. As a matter of fact H_0 , H_1 and the diameter distribution determine the height curve completely.

The functions are tested against the independent testing material. Results are given in chapter IV. These tests reject my functions. Judgement should not depend solely on these tests. The Chi-square test will reject any hypothesis if the number of observations is high enough. A judgement must be made from a forestry point of view. To do this several plots are studied. Figs. 6, 7, 8, 9 show the diameter distribution accumulated over all stands. Figs. 10, 11 show the mean residual between observed and calculated heights. In appendix 3 plots of single stands are given. From these plots we have concluded that the functions have an acceptable fit.

Diameterfordelinger og høydekurver for ensaldrede bestand av *Pinus sylvestris L.*

Hensikten med dette arbeidet er å lage funksjoner som beskriver diameterfordelingen og høydekurven til bestand av ensaldret furu. Bestandsparametre er å betrakte som kjente og funksjonene kan basere seg på disse. Et antall kjente bestand er grunnlag for beregningene. Materialet er beskrevet i kapittel II. Materialet omfatter både tynninger og stående bestand. Materialet er blitt delt (tilfeldig) i to grupper, beregningsmateriale og testmateriale, henholdsvis. Funksjonene utvikles på beregningsmaterialet. Disse testes så mot det uavhengige testmaterialet.

Johnsons system b fordeling (J-Sb) er brukt for å tilpasse diameterfordelingen. J-Sb er en fire-parameterfordeling ((III.A.1), (III.A.3)) som kan variere skala, lokasjon og form. J-Sb er en transformasjon av en normalfordelt variabel. Det område av «former», målt i skjevhet og kurtosis som J-Sb kan oppnå dekker det område som diameterfordelingene hovedsakelig befinner seg i (Fig. 2, 3, 4 og 5).

Vi har brukt sannsynlighetsmaksimeringsprinsippet ved estimering av parametrene på hvert bestand. Variasjonene fra bestand til bestand er håndtert i en multippel regresjon, der bestandsdata er uavhengige variable. J-Sb parametrene er ikke brukt direkte, de er transformert til faste fraktiler (III.B.2) som brukes som avhengige variable i (III.B.3). Regresjonskoeffisienter er gitt i tabell 3. For å finne fordelingen på en bestemt bestand, må vi først beregne fraktilene (III.B.3) (tabell 3) så transformere tilbake til J-Sb

parametrene ved (III.B.6). (En slik «tilbake transformasjon» kan ikke alltid utføres. (III.B.4) og (III.B.5) gir betingelser for å kunne gjøre det.) Fordelingen beregnes så ved (III.A.3).

D_g inngår som uavhengig variabel i regresjonsberegningene. D_g er også en egenskap ved fordelingen. Dette er det ikke tatt hensyn til under beregningene. Det har vist seg at den beregnede D_g ikke avviker mye ifra den oppgitte.

Kurven gitt ved (III.C.1) er brukt som høydekurve. Da H_0 og H_1 skal betraktes som kjente, burde kurven oppnå disse mål på hvert bestand. En algoritme er utviklet slik at det vil skje. Faktisk vil H_0 , H_1 sammen med den funne diameterfordelingen bestemme høydekurven fullstendig. Kurven avhenger derfor ikke av noe observert høydemateriale.

Testresultater mot det uavhengige testmaterialet er gitt i kapittel IV. Testene er stort sett signifikante, dvs. materialet motsetter seg funksjonene. Kji-kvadrat test er dårlig egnet i slike situasjoner da den vil forkaste enhver hypotese bare vi har mange nok observasjoner. Det er nødvendig å foreta en «skoglig» vurdering av resultatene. Selvom det er «signifikant» forskjell mellom teori og virkelighet, hvor stor, hvor viktig er denne forskjellen? Ut fra slike betraktninger er metoden funnet tilstrekkelig god. I fig. 6, 7, 8 og 9 har vi summert diameterfordelingene for hele materialet. Dette vil vise hvor godt funksjonene treffer midlet over en hel «skog» med den sammensetning vårt materiale har. I fig. 10 og 11 er gjennomsnittlig avvik mellom observerte og beregnede høyder skrevet ut for hver diameterklasse. De beregnede høyder synes «for store» for de høyde diameterklasser. Dette kan henge sammen med at diameterfordelingen (og dermed plasseringen av H_0) er dårlig bestemt i denne enden. I appendiks 3 finnes plot over en del enkeltfelter.

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Appendix 1

- T_{1.3} Age at breast height (years)
Alder ved brysthøyde (år)
- H₀ Top Height, arithmetic mean height of 100 largest (according to diameter) trees per hectare (m)
Overhøyde. Aritmetisk middel av de 100 grøvste trær per hektar (m)
- N₃ Number of standing trees after thinning (per hectare)
Antall trær etter tynning pr. ha
- H₃ H (Lorey) after thinning (m)
Grunnflateveid middelhøyde (m) etter tynning
- D₃ Basal area mean diameter after thinning (cm)
Grunnflatemiddelstammens diameter etter tynning (cm)
- G₃ Basal area after thinning (m² per hectare)
Grunnflatesum etter tynning (m² pr. ha)
- V₃ Volume after thinning (m³ per hectare)
Volum etter tynning (m³ pr. ha)
- N₂ Number of removed trees (thinnings) (per hectare)
Antall uttatte trær (tynning) pr. ha
- H₂ H (Lorey) of removed trees (thinnings) (m)
Grunnflateveid middelhøyde, tynning (m)
- D₂ Basal area mean diameter of removed trees (thinnings) (cm)
Grunnflatemiddelstammens diameter, tynningsuttak (cm)
- G₂ Basal area of removed trees (thinnings) (m² per hectare)
Grunnflatesum, tynningsuttak (m² pr. ha)
- V₂ Volume of removed trees (thinnings) (m³ per hectare)
Volum, tynningsuttak (m³ pr. ha)
- H₁ H-Lorey, either H₂ or H₃
enten H₂ eller H₃
- D_g either D₂ or D₃
enten D₂ eller D₃
- H₄₀ Site index. H₀ at T_{1.3} = 40 years (m)
Bonitet. H₀ ved T_{1.3} = 40 år (m)
- n₂ Actual number of thinned trees
Faktiskt antall tynnede trær
- n₃ Actual number of standing trees
Faktiskt antall stående trær
- N Either N₂ or N₃
Enten N₂ eller N₃
- n Either n₂ or n₃
Enten n₂ eller n₃
- ˆ This symbol above any parameter denotes «estimate of»
Skrevet over et tegn betyr «estimat»
- d_α A diameter where α · 100 % of the trees have smaller diameters
En diameter slik at α · 100 % trær er tynnere
- Φ Cumulative standard normal distribution
Kumulative standard normalfordeling

J-Sb Johnson System b distribution

Johnson System b fordeling

ξ Location parameter in J-Sb (ξ equals D_{\min})

Lokasjonsparameter i J-Sb (ξ lik D_{\min})

λ Scale parameter in J-Sb ($\xi + \lambda$ equals D_{\max})

Skalaparameter i J-Sb ($\xi + \lambda$ lik D_{\max})

γ, δ These parameters in J-Sb determines the «form» of the distribution.

Formparametre i J-Sb

β_1 Skewness

Skjevhet

β_2 Kurtosis

Kurtosis

d Diameter of a single tree

Diameter på et enkelt tre

h Height of a single tree

Høyde på et enkelt tre

g Basal area of a single tree

Grunnflate på et enkelt tre

T Vector of some stand parameters

Vektor av bestandsparametrene

P(.) Probability

Sannsynlighet

Appendix 2

Maximum likelihood estimation of J-Sb parameters

Consider observations d_1, \dots, d_m stochastically independent realisations from a J-Sb.

In this material the m diameters are grouped into 2 cm classes. We know the class frequencies and also the class means. This involve somewhat more information than pure grouped data. LAMBERT (1970) has estimated J-Sb parameters with maximum likelihood of grouped and ungrouped data (as if they were ungrouped). He concluded that the grouping did not affect the estimates much. Here the data are used as if they were original diameters. (Each class mean is weighted with the class frequency.)

The log likelihood is

$$\ln(f(d_1, \dots, d_m)) = -\frac{m}{2} \ln(2\pi) + m \ln \delta + m \ln \lambda - \sum_{i=1}^m \ln(d_i - \xi) - \sum_{i=1}^m \ln(\lambda + \xi - d_i) - \frac{1}{2} \sum_{i=1}^m \left[\gamma + \delta \ln \frac{d_i - \xi}{\lambda + \xi - d_i} \right]^2$$

to ease notation set

$$\ln_i = \ln \frac{d_i - \xi}{\lambda + \xi - d_i}$$

Differentiation of log likelihood yields equations

$$\hat{\delta} = \sqrt{\frac{m}{\sum \hat{\ln}_i^2 - \frac{(\sum \hat{\ln}_i)^2}{m}}}$$

$$\hat{\gamma} = \frac{\hat{\delta} \sum \hat{\ln}_i^2}{m}$$

where estimates of ξ, λ have been inserted in $\hat{\ln}_i$. This is as usual with normal distribution. (\ln_i then being the observations.) To solve for ξ and λ the following equations must be solved:

$$\frac{\partial \ln(f(d_1, \dots, d_m))}{\partial \xi} = 0$$

$$\frac{\partial \ln(f(d_1, \dots, d_m))}{\partial \lambda} = 0$$

where $\hat{\gamma}$, $\hat{\delta}$ are inserted after differensiation. These equations may be solved by Newtons method.

Starting values of ξ and λ was

$$\xi_0 = d_{\min} - \Delta$$

$$\lambda_0 = d_{\max} - \xi_0 + \Delta$$

were

$$\Delta = \frac{d_{\max} - d_{\min}}{\sqrt{m}}$$

My motivation is as follows:

$$\text{Define } f_i = (1 + \exp\left[\frac{\gamma - \phi^{-1}\left[\frac{i}{m+i}\right]}{\delta}\right])^{-1} \quad i = 1, \dots, m$$

Now

$$\begin{aligned} E(d_{\min}) &\approx \xi + \lambda \cdot f_1 \\ E(d_{\max}) &\approx \xi + \lambda \cdot f_m \end{aligned}$$

(The inverse image (J-Sb) of expectation of ordered observations from uniform distribution.)

Solving the above equation with expectation removed gives

$$\xi = d_{\min} - \frac{(d_{\max} - d_{\min})}{\frac{f_m - f_1}{f_1}}$$

$$\lambda = d_{\max} - \xi + \frac{(d_{\max} - d_{\min})}{\frac{f_m - f_1}{1 - f_m}} = \frac{d_{\max} - d_{\min}}{f_m - f_1}$$

f_i is dependent of δ , γ . Setting $\delta=1$, $\gamma=0$ then

$$\frac{f_m - f_1}{f_1} = \frac{f_m - f_1}{1 - f_m}$$

and the f_1 's are independent of parameters.

$\frac{f_m - f_1}{f_1}$ is fairly approximated by \sqrt{m} as the table shows:

Table appendix 2.

m	\sqrt{m}	$\frac{f_m - f_1}{f_1}$
10	3.16	2.76
20	4.47	4.17
30	5.18	5.12
40	6.32	6.1
50	7.07	6.8
75	8.66	7.8
100	10.00	9.2
500	22.6	16.7
1000	31.6	20.9

Better «fit» may be achieved by other exponents, if of interest.

In some observations the process (Newton) did not converge. This may be due to the existence of a path of the likelihood on which it is unbounded (LAMBERT, 1970). For these observations the starting values of ξ , λ were used. The stopping criterion used was to stop at stage k if

$$\frac{(\lambda_k - \lambda_{k-1})^2 + (\xi_k - \xi_{k-1})^2}{\lambda_{k-1}^2 + \xi_{k-1}^2} \leq 0.0001$$

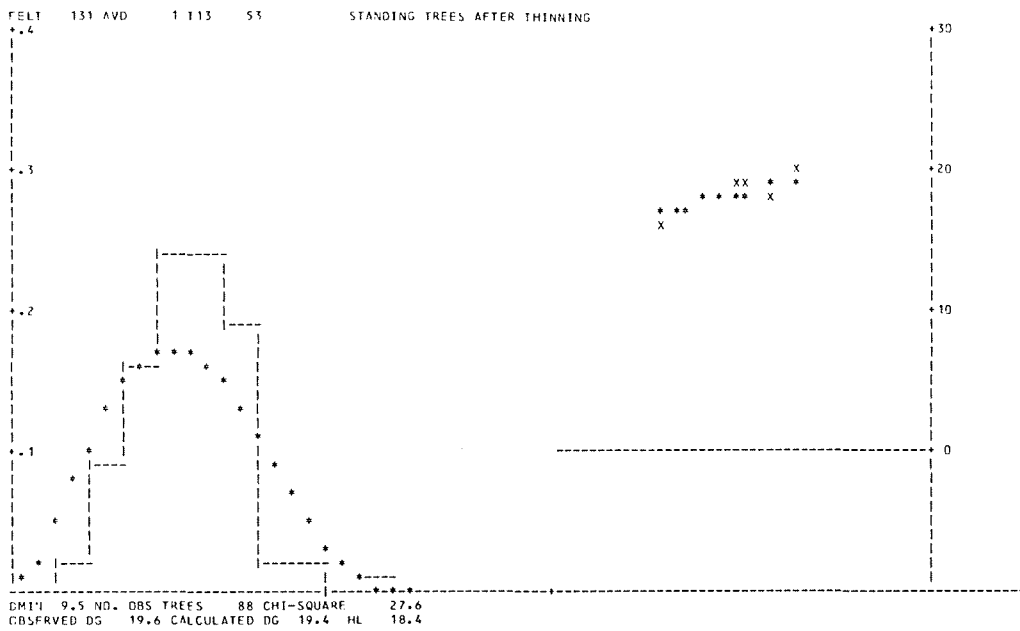
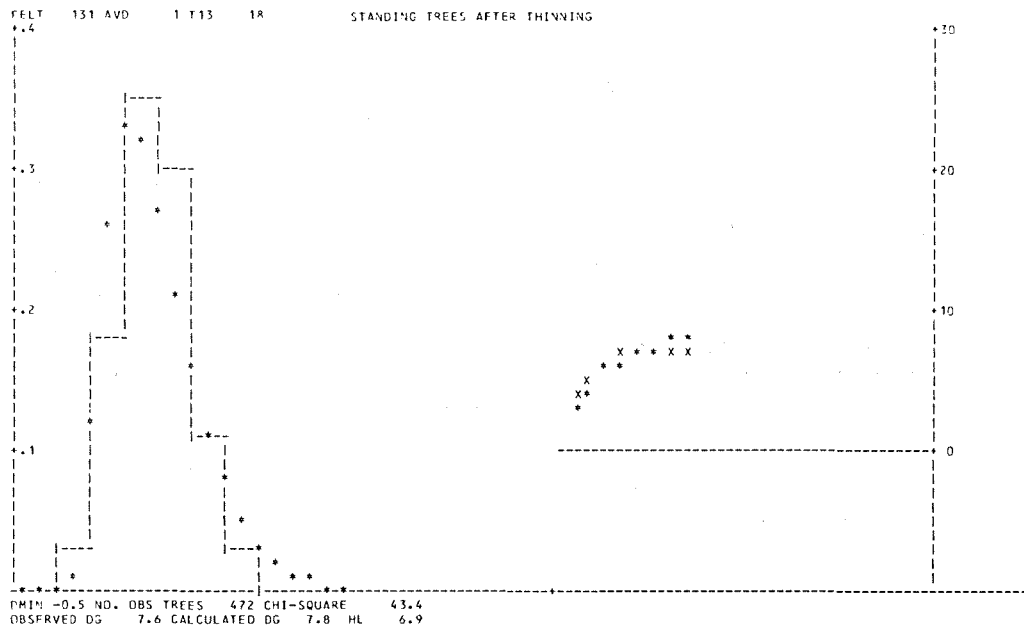
k was also limited to 50. Usually the process stopped after about 3—4 iterations.

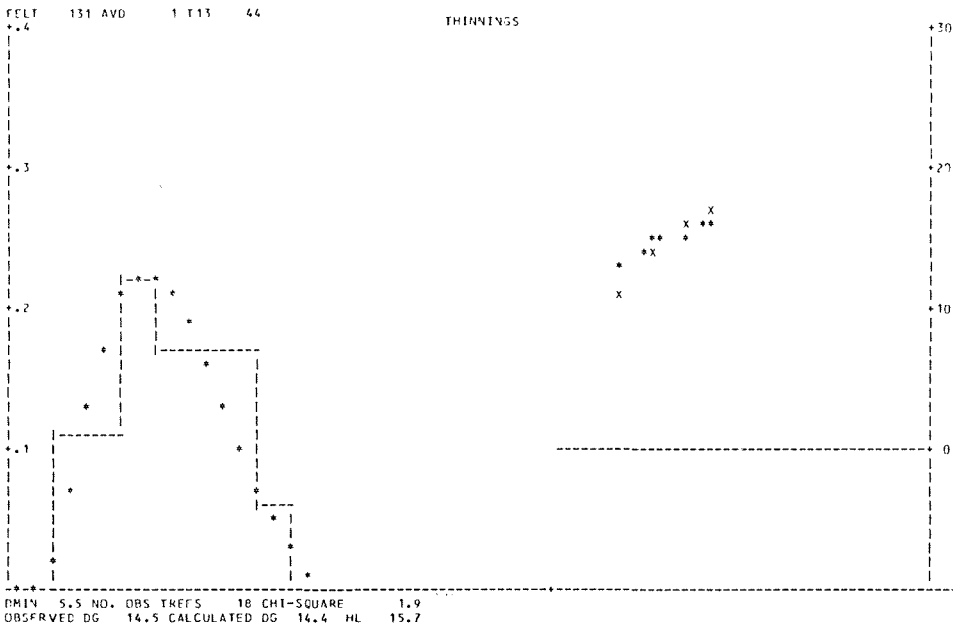
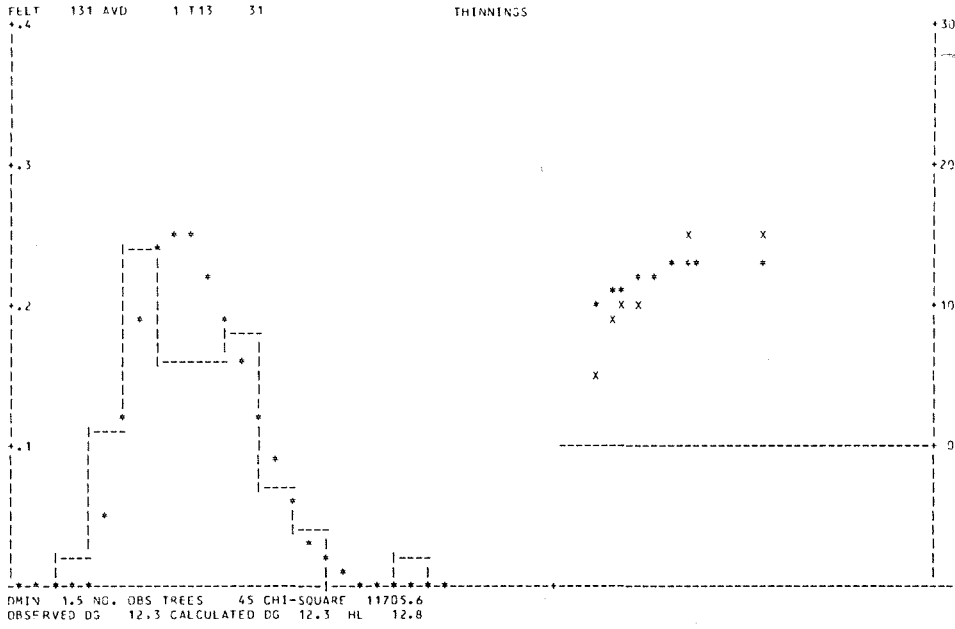
Appendix 3

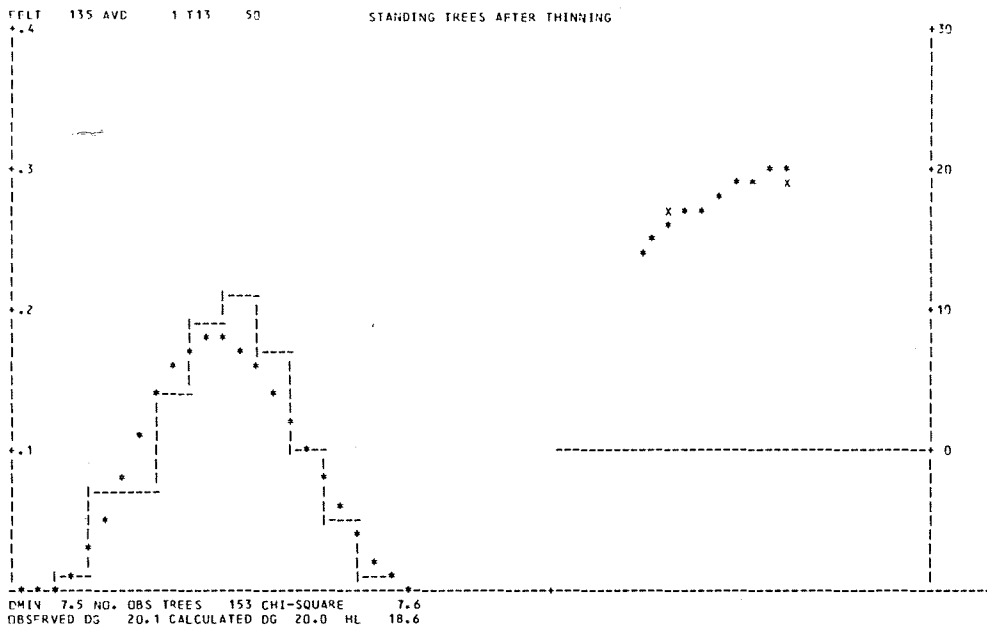
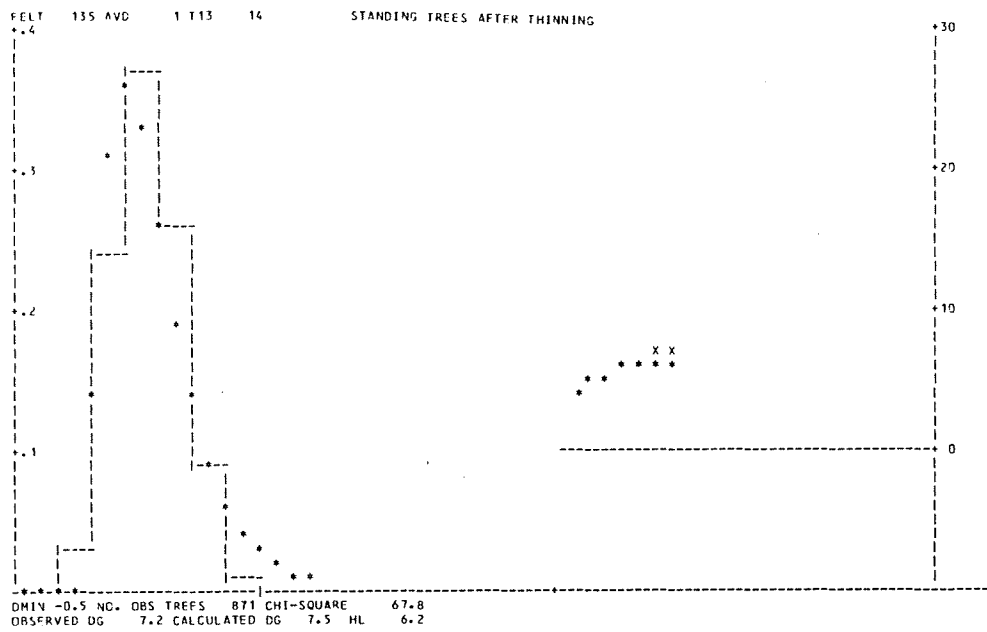
Plots of diameter distributions and height curves on some specific stands at specific ages

The plots are computer made. The plot consist of both the diameter distribution and the height curve. The histogram is the observed diameter distribution (2 cm classes). The «*» curve is the distribution calculated from the functions. (The abscissa is the diameter, the ordinate is the fraction of trees. The «DMIN» indicate the diameter at the «origo» of the plot.) The right part of the plot concerns the height curve. The «X» indicate observed height, the «*» is the calculated height curve. if the deviance between observed and calculated height is less than 1 m no «X» is plotted. (The abscissa is the diameter, the ordinate is the height in meters.) Also the no. of observed trees, the Chi-square test, the H_1 , the observed and the calculated D_g is given.

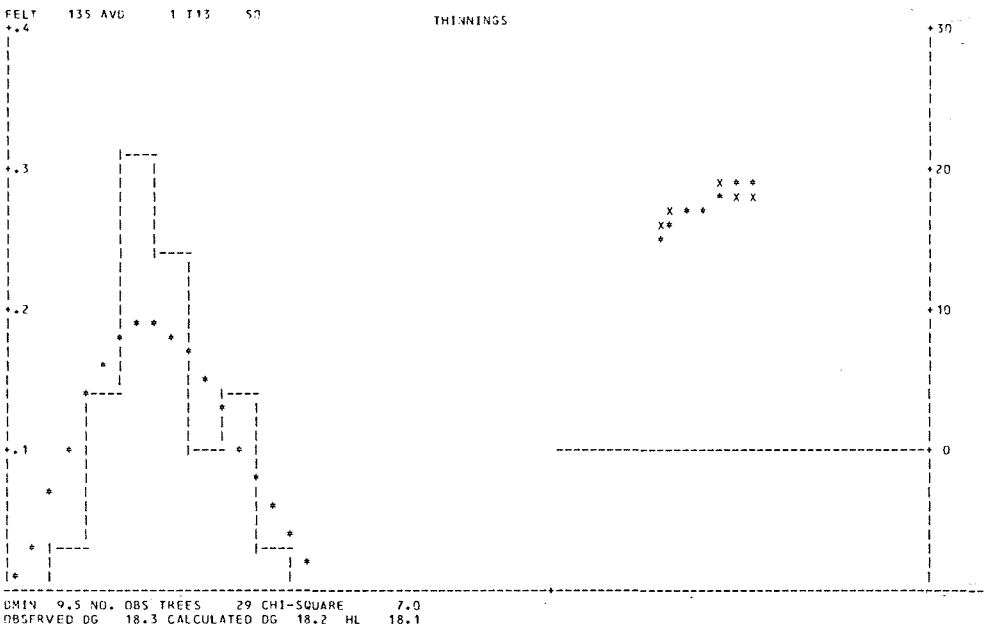
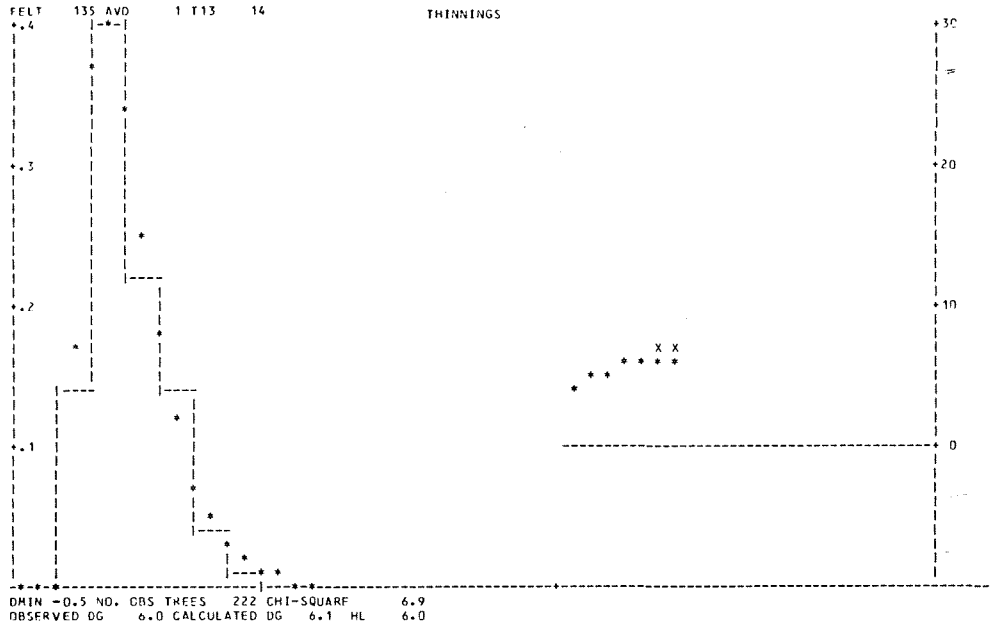
These plots show how the functions fit to an independent set of observed stands. One might get an idea on how the functions work without the need to know all the mathematical background.

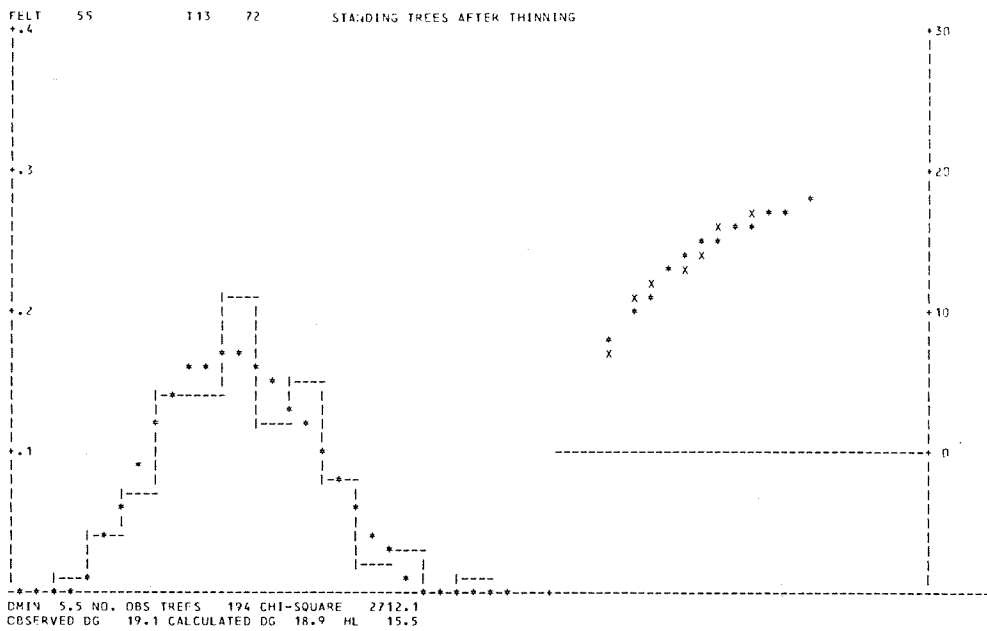
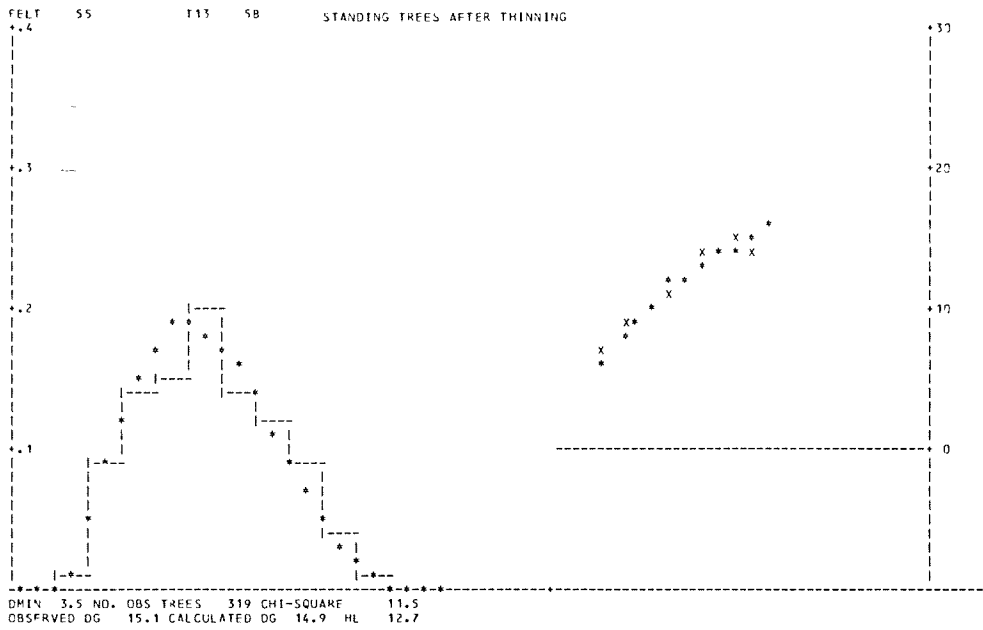


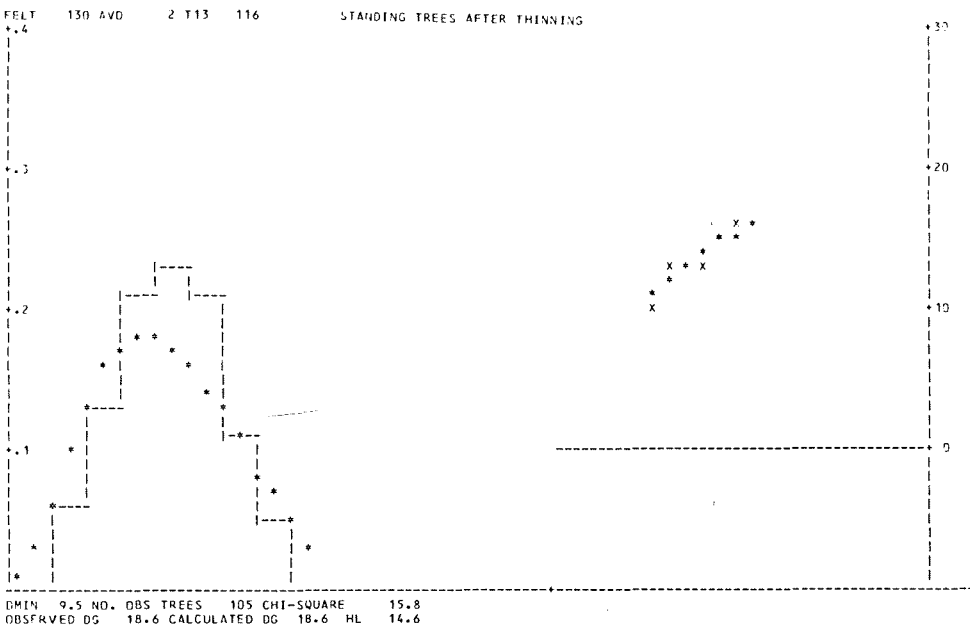
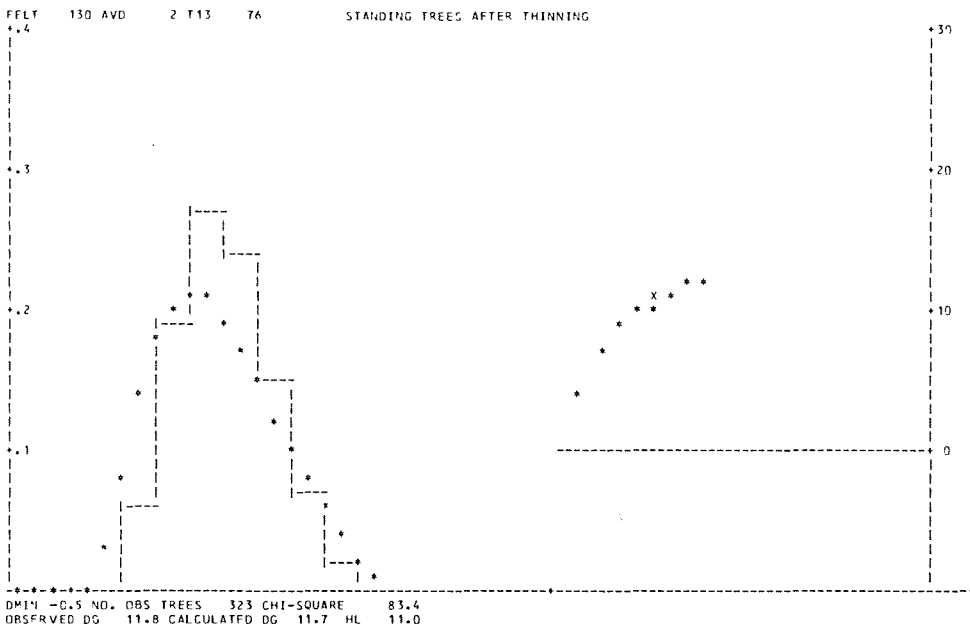




*Diameter distributions and height curves
in even-aged stands of Pinus Sylvestris L.*







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