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Power Laws and Market Shares: Cumulative Advantage and the Billboard Hot 100

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Hi-res EPS files of equations and figures: https://figshare.com/s/45ea9d074b5927eafbf4

Abstract

The existence of highly skewed success distributions in the music industry has been repeatedly demonstrated by scholars, but there still is no agreement about how these shapes relate to concepts like 'talent', 'reputation', and 'quality'. Starting from the theories of Rosen (1981) and Adler (1985), this article concentrates mainly on the phenomenon referred to as cumulative advantage, as one of the leading candidate mechanisms to explain the formation of the 'power law-like' distributions found in e.g. the sales of music recordings. We make the case for the pivotal role of the market share approach in the music industry and demonstrate its efficacy as a 'success measure' methodology by providing a descriptive summary with regard to 'connotations' of cumulative advantage based on fifty years of *Billboard* Hot 100$^{©}$ history. Our results indicate that, while records that have sold well will keep on selling, the same might not be true for recording artists. However, a modest 'star power' effect may have represented a small but vital edge for the oligopoly of multinational recording companies. The methodology suggested in this article should provide students of 'hit song science' and the likes with a more rigorous approach to appraising commercial success, as well as a comprehensive background as to its origin and relevance to popular music studies.

Keywords

Billboard Hot 100, economics of superstars, cumulative advantage, market share, power laws, hit song science

1. Introduction

In 2012, the Super Bowl halftime show featured the American entertainer Madonna. Her performance set a new record for the most-watched Super Bowl halftime show in history, and included the premiere of her then-new single 'Give Me All Your Luvin'' (Gallo, 2012) which debuted at #13 on the *Billboard* Hot 100© for the week ending February 18 and climbed three places the following week, edging into the coveted top ten. Abruptly, the song fell 29 spots in just one week, landing at #39; the next week, it tumbled almost 20 more places to #58, the next, all the way down to #86, and then it dropped out of the Hot 100. Ultimately, Madonna's Super Bowl song failed to make it into the *Billboard Year-End Hot 100 singles of 2012*, and for comparison, the recording that topped that chart, a song called 'Somebody That I Used to Know' by the previously relatively unknown Belgian-Australian artist Gotye, spent 59 weeks in the Hot 100 and eight weeks at the #1 spot.

What might explain the fate of Madonna's song? This article aims to shed light on this question while simultaneously providing a comprehensive – and comprehensible – review of the 'economics of superstars' that is intended for a non-economics audience with a scholarly interest in popular music.

1.1 'Sound recording popularity charts: a useful tool for music research'

Dr. Peter T. Hesbacher (1937-2015) was an associate professor of sociology and psychiatry at the University of Pennsylvania. His other lifelong passion was pop music. He had a collection of more than 100,000 records and did statistical analysis of record trends for *Billboard* magazine (Smith, 2015). Hesbacher contributed a substantial number of academic articles on popular music and some notable works have recently been subject to a timely revisit by Carroll (2015), which serves as a convenient point of departure for the current article.

Since Hesbacher's call for increased scholarly attention to sound recording popularity charts (e.g. Hesbacher, Downing, & Berger, 1975b, p. 86), such data have been utilized in various areas of scientific research; by researchers of geography and

public policy (e.g. Scott (1999)), psychology (e.g. DeWall, Pond Jr, Campbell, and Twenge (2011)), and 'music information retrieval' (e.g. Herremans, Martens, and Sörensen (2014); Mauch, MacCallum, Levy, and Leroi (2015)), but arguably not to the degree commensurate with the potential envisioned by Hesbacher. If we assume that popularity charts in some way reflect an evolving common culture, these historical documents have the capacity to serve research as a measurement variable that is generally both objective and quantitative. The primary objective of this article is to demonstrate a simple yet powerful way to operationalize this variable. This article will focus exclusively on the American *Billboard* Hot 100, which, according to Bradlow and Fader, '… has had a dramatic impact on the music industry and American culture' (2001, p. 369).

1.2 Data collection considerations

When using song charts such as the Hot 100 as empirical data, researchers confront additional issues related to the elusiveness and transience of the chart compiling methodologies. In short, *Billboard* has historically derived its chart rankings from two major factors: radio play and record sales. Its data collection in this regard was traditionally survey-based, in which each weekly sample was obtained by calling a selection of sales outlets and radio stations (Hesbacher, Downing, & Berger, 1975a). This rather clumsy and subjective arrangement was obviously prone to various forms of error and manipulation, a presumption that was verified by the 'SoundScan Revolution' that introduced more reliable 'direct observation' data from barcode-reading sales registers (Anand & Peterson, 2000). As technology and formats have evolved, new measurement parameters have also been added, e.g. data from streaming services like Spotify$^{©}$ were included in 2007, and even YouTube$^{©}$ views have been taken into account since 2013.

Billboard magazine occasionally publishes notes on its chart compilation methodology, but neither the collected field data nor the exact formulas that translate the data into chart rankings are available. Therefore, it has never been possible to independently evaluate or replicate *Billboard*'s procedures. The Hot 100 is unquestionably a widely accepted picture of reality in the US music industry. In fact, as a window on the world, record labels have tended to rely more on the *Billboard* charts than on their own sales (Anand & Peterson, 2000). In this manner, although the data

might be considered 'contaminated' to a certain degree, the Hot 100's 'validity' as a measurement of commercial success is inarguably high.

1.3 The relative and the particular

The Hot 100 only declares how the selected participants rank by popularity among themselves; it provides no information on *how much more* popular one song is over another. Statisticians often refer to measures like ranks (i.e. chart positions) as *ordinal*. Compared to the more versatile 'continuous' *ratio* variables (e.g. 'length in inches'), data in the form of rank numbers have certain built-in constraints. It does not make much sense to add or subtract chart positions or to calculate the mean or standard deviation of a set of values, and the number 0 is not particularly meaningful as a rank.

To be sure, there are particular statistical formulas and procedures for ordinal data, and dedicated Hot 100 studies have even introduced approaches that are novel enough to be published in high-ranking statistical journals (e.g. Bradlow and Fader (2001)). However useful these methods may be, they can never transform the relative property of a chart position into something more expressive or answer simple questions such as, *'what is better, spending two weeks at #3 or one week at #1?'*

1.4 Comparing chart performance: the J-curve

Carroll (2015) presents a new methodology to compare different records' chart performances, e.g. two weeks at #3 versus one week at #1, and reviews a range of earlier contributions that have similar objectives, including Hesbacher, Anderson, Snyderman, and Koppel (1982). The basic concept of most of these procedures is to assign different 'weights' or 'scores' to the respective chart positions that are meant to better reflect the underlying 'field data', i.e. record sales and airplay figures. Both Carroll (2015) and Hesbacher et al. (1982) provide formulas that take a weekly chart position as input and return an appropriately weighted value.

According to Carroll, 'Hesbacher had unique access to the *Billboard* methodology as a consultant, so one suspects this scheme may reflect the field data results. He fitted these weights to an equation he called a "J-curve"' (2015, p. 595). When plotted as a graph, the different values it produces across the 100 chart positions

do not form a straight line and instead take the shape of a sloping curve. The term 'J-curve' is not exclusive to Hesbacher et al. and is used in several different fields to describe very different phenomena. Mathematicians often refer to similar-looking distributions as 'leptokurtic' or 'stable Paretian', or even 'highly skewed' or 'fat-tailed'.

A most important point here is that the J-curve also carries a deep and non-trivial message: the market for popular songs is an economy of stark inequality in which rewards are disproportionally concentrated among the higher-ranking participants, i.e. the stars. Evidently, this description also fits a range of other human activities and Hesbacher et al.'s equation is thus apparently closely related to the more famous 'Pareto principle', a.k.a. 'the 80–20 rule', as in '80% of the land in Italy is owned by 20% of the people'.

The existence of highly skewed demand distributions in the music industry has since been demonstrated repeatedly and for other sub-sectors like concerts (Krueger, 2005); eventually, with the advent of the rich data flow from digital distribution platforms like iTunes©, these J-curves have been depicted with high degrees of accuracy (see e.g. Duch-Brown and Martens (2014, p. 10)).

1.5 Structure of the article

Section 2 opens with a brief review of the origins of the scholarly field known as 'the economics of superstars' and then concentrates mainly on subsequent literature with reference to the concept of cumulative advantage. We also provide an overview of empirical research that have tested such models using data from music recording popularity charts. The case is made for the pivotal role of the market share approach in the music industry, and how the distribution of market share relates to power laws. The chapter concludes with a suggested methodology for applying these principles. Section 3 presents a descriptive summary of fifty years of Hot 100 data, where the main objective is to demonstrate the approach and methodology presented in the previous chapter, while also providing a data rich overview that allows for informal reflections with regard cumulative advantage theories and a considerable span of popular music history. Section 4 highlights figures from the summary that are considered particularly interesting for theories of cumulative advantage, and different interpretations of the data

are discussed. The final sections 5 and 6 offer concluding remarks and suggestions for future research.

2. Theories of superstardom

Neither Carroll nor Hesbacher et al. dwell much on the deeper meanings of the J-curves – neither asks *why* or *how* they arise – but these particular shapes have puzzled scholars for more than one hundred years. In the context of the entertainment industry, seminal contributions were made in the 1980s by two American labour economists, Rosen (1981) and Adler (1985). The former actually introduced the term 'superstar' to economists' vocabulary and provided a much-quoted definition of the phenomenon 'wherein relatively small numbers of people earn enormous amounts of money and dominate the activities in which they engage' (Rosen, 1981, p. 845).

Rosen and Adler are typically portrayed as proponents of mutually exclusive and competing theories in which the dichotomy criterion denotes whether there is a relationship between talent and success. We believe that 'talent' here should be interpreted in a broad sense; e.g. to be in possession of a quality that represents an advantage, also of the kinds that would not be considered purely artistic. What is not questioned is Rosen's claim that certain conditions must exist for superstars and J-curves to arise: the activity must target a sizable market, it must have media attention, and it must also lend itself to reproduction – preferably in a perfect and endless manner – which again enables mass production and consumption. Obviously, a dentist's practice can hardly satisfy these requirements, but the present-day pop star who sells her recordings digitally over the Internet does so with an effectiveness that must have been difficult to envision only two decades ago (cf. Rosen, 1983, p. 460).

Rosen's theory is known as an equilibrium model – a popular device in neoclassical economics – and as with all such schemes, it is a stark simplification of the reality it aims to describe. The model is qualitative and rather an aggregate function – the summation – of the behaviours of all the individual participants in the market, including consumers and performers. The basic version assumes that the paying audience is a large, homogeneous mass; it is extended to a two-tiered group later in the paper, a simplification that probably does not excite music and culture scholars. In terms of equilibrium, this 'system' seeks to achieve 'balance', which is established when all participants are content with their actions and feel no reason to change them; thus, the price changes until both sellers and buyers find it acceptable.

When the system finally rests, the result is a J-curve; a proportionally small group of performers takes home the lion's share of the revenues, leaving only 'crumbs' to the remaining majority. The main assumption of Rosen's theory is that this result occurs because lesser talent is a poor substitute for greater talent in the entertainment industry and in other similar contexts. Watching two shows by an artist of average talent does not equal the satisfaction of experiencing one performance by a superstar; similarly, a single song of superstar quality is infinitely more valuable than a series of many mediocre songs. Notably, this assumption seems particularly apt in the context of the recording industry, which has a long-standing tradition in the peculiar practice of uniform pricing, e.g. '99 cents for all tracks on iTunes' (Shiller & Waldfogel, 2011). Thus, there is no economic incentive for consumers to choose the lesser talent or quality, as it is not even offered at a discount. However, in Rosen's model, there is no absolute requirement for superstardom, you simply must be that bit more talented than the other performers. As summarized by Adler (1985), 'persons with only a slightly greater talent command much higher incomes than those who are only slightly less talented; output is concentrated on those few who have the most talent' (p. 208). Thus, according to Rosen's theory the artist that ends up with the largest pile of money is the individual whose talent is greater than the rest by just the right amount.

Rosen does not describe the features of the talent distribution, nor does he explain how this distribution came to exist – it is completely arbitrary. Moreover, differences in talent are known and agreed upon by all consumers – yet another discussible assumption – and superstars can therefore emerge in a straightforward manner. Although he considered only the performing arts, MacDonald (1988) introduced the stochastic (i.e. random or non-deterministic) process into Rosen's framework, in which 'talent' determines the difference in probability that a particular performance will be good. The quality of a good or bad performance is the same for all artists; however, the probability of putting on good shows is 'serially correlated'. In other words, the artist's record of accomplishment affects the likelihood of future success; hence, there is a substantial chance that a performance by a superstar talent will be satisfying and that she or he will therefore be able to charge higher ticket prices and attract larger audiences. Thus, MacDonald's process, which is a bit more involved than is described here, will also eventually result in a J-curved income distribution.

To the contrary, Adler (2006) argues that the emergence of J-curves and superstars is not explained by the differences in the talent or quality of the artistic offerings but instead by the peculiar nature of how such products are consumed. 'The consumption of a piece of art is not a momentary experience but a dynamic process in which "the more you know, the more you enjoy"' (p. 3), a phenomenon psychologists call 'mere exposure' (Zajonc, 1968). Therefore, because knowledge is required to realize the value of art products, consumers will seek products from which this knowledge can be most easily acquired. In econ-speak, when enjoying the music of specific artists, 'consumption capital' is accumulated (Stigler & Becker, 1977), which results in increasing 'marginal utility', the additional amount of satisfaction that is derived from consuming an additional unit of a product. Hence, superstars arise because economizing consumers attempt to minimize their search costs and accumulate consumption capital as effectively as possible.

Acquiring such knowledge takes three different forms: exposure to the art itself, interaction with other people, and/or media. When an artist is popular, it is easier to find discussants who are familiar with the artist or to find media coverage; hence, consumers prefer to consume what others also consume. Because the number of artists who can be popular at any one time is limited, not all talented artists can be successful. For an economizing music consumer, already popular artists represent the best deal and given a choice between two artists of similar talent, a consumer is better off choosing the one his or her friends have already chosen. Similarly, when given a choice between two records of similar quality, a consumer is better off buying the one from the artist he or she already knows. Therefore, whoever happens to have a head start in this game, by random luck, will 'snowball into' a star and become a member of the 'fat tail' of the resulting J-curve. In other words, superstars can rise among performers of equal talent.

Frank and Cook (1995) introduced the mainstream public to the economic concept of J-curves in their non-fiction bestseller, *The Winner-Take-All Society*; the book title itself is a meaningful reference to the reward structure of markets in which superstars dominate. The superstar phenomenon is of course also inextricably linked to more general perspectives on income inequality. The book can be regarded as a critical commentary on the economic and social consequences of this phenomenon on societies in which increasingly more individuals compete for increasingly fewer, but increasingly greater, rewards. There are several more comprehensive reviews of this literature

stream, see e.g. Adler (2006) and Schulze (2011). As the former posits, '[t]he Economics of Superstars sets out to explain the relationship between talent and success in the arts, but there is no agreement about what this relationship is' (Adler, 2006, p. 2).

It has been enthusiastically prognosticated that, as consumer search costs are lowered by (among other things) internet features such as search, recommendation, and filtering tools, the new digital platform will benefit lesser known and newer artists and therefore may even erode the reign of the superstars (e.g. Brynjolfsson, Hu, & Simester, 2011). However, according to Elberse (2013), sales data from the digital music services do not support this assumption; in fact, they exhibit a trend that is the opposite; the concentration is increasing, i.e. the most popular songs hold an even larger share of the market than before (p. 116). This observation, if correct, is evidently of some significance to the Rosen/Adler debate, but consequent theorising is still in its infant stage.

2.1 Cumulative advantage: the effect of many names

Adler's model belongs to a large and somewhat disorderly family of 'processes' often referred to as cumulative advantage mechanisms. The 'cumulative' component can be traced back to the Swedish scholar Gunnar Myrdal, and, as far as we are aware, 'advantage' was added by Derek J. de Solla Price, as described below. In what is perhaps the most extensive review undertaken by sociologists, DiPrete and Eirich (2006) present cumulative advantage as a 'general mechanism for inequality across any temporal process … in which a favourable relative position becomes a resource that produces further relative gains' (p. 271). Numerous other names have been given to 'effects' that fit this description, e.g. 'Matthew effect' (Merton, 1968), to name just one.

The important point here is that cumulative advantage is arguably the leading candidate mechanism to explain the formation of J-curves and superstars, and both the inequality and unpredictability of success, as a result (Watts, 2007). In the current setting, we will attempt to observe how advantage begets further advantage in that 'a song that has already sold many copies will keep on selling many copies' or 'an artist that has already sold many records will keep on selling many records'. To illustrate further, we invoke three 'main connotations' of cumulative advantage that DiPrete and Eirich (2006) identify in the literature (p. 10):

1. The rate of growth in an outcome variable is a function of the current values of that outcome.
2. Small advantages at an early stage of a process grow larger over time.
3. Inequality grows over time as a consequence of the cumulative advantage process.

2.2 Modeling cumulative advantage

Models in which 'new objects tend to attach to popular objects' are now often referred to as 'preferential attachment' models (see Barabási and Albert (1999)), which is arguably the current designation for cumulative advantage and perhaps most eagerly studied by scholars of network science. Obviously, a mechanism of such straightforward description will lend itself to mathematical modelling, and the first version was introduced by Eggenberger and Pólya (1923) and was soon followed by a second version offered by Yule (1925). The latter used his version to explain the distribution of species among the genera of plants and animals, as reported by John C. Willis' statistical studies of biological taxonomy – which indeed included some nice graphs of J-curves. Notably, a year later, Lotka (1926) showed that a completely different 'animal', namely, the frequency distribution of scientific productivity (as in the number of authors sorted by the number of published articles they have to their name) also takes the shape of a J-curve, which has since been referred to as Lotka's Law – the first 'law of scattering'.

Yule and Lotka's J-curve phenomena were later subject to extensive studies, perhaps most notably *Yule-Simon* (Simon, 1955) and the *Pólya urn* (Price, 1976). The latter is what is known as a stochastic urn process, typically just a pictorial description, in which balls are added to a set of urns based on a set of rules and/or random mechanisms. In Price's words, '[i]n general, the model supposes that fate has in storage an urn containing red and black balls; at regular intervals a ball is drawn at random, a red ball signifying a 'success' and a black ball a 'failure'' (1976, p. 293). One rule can then for instance be to add another ball of the same colour to the urn. In another simple variant of a Pólya scheme, additional balls are continuously distributed among a collection of urns as function of the number of balls the urns already contain. A Yule-Simon urn scheme requires that the number of urns also increases continuously, but this is not a necessary condition for preferential attachment per se, and a decreasing number

of urns is even possible. Thus, a Yule-Simon process applied two our setting will rests on the two following assumptions: the probability that consumer $n + 1$ chooses a record already chosen by exactly k of the previous n consumers is proportional to k; and that there is a constant probability $0 < p < 1$ that consumer $n + 1$ choose a record not previously chosen by anyone.

The major point that Price and others have made is that simple cumulative advantage processes such as these urn schemes generate J-curves, and similar-looking distributions such as Yule, Lotka, and Pareto are indeed all related. As these shapes occur in an intriguing variety of both 'man-made' and natural systems, this theory has quite a bit of 'conceptual significance' to it (Price, 1976, p. 304). Accordingly, the Yule process is now the most widely accepted theory to explain certain distributions, such as academic citations, city populations, and personal income (Newman, 2005).

2.3 Models of superstardom: empirical testing

Thus, if the distribution of success among pop artists can be 'replicated' by a simple random urn scheme, is 'real world' success random, too? This conclusion would evidently support Adler's theory that stardom does not require superior talent; instead, it requires only a stroke of luck. Arguably, observing an appropriate J-curve in empirical data might indicate a cumulative advantage effect and, hence, not finding one might indicate no effect, but the opposite does not necessarily hold. Evidently, a J-curve is not sufficient proof that a cumulative advantage mechanism is at work; it is, however, the basic strategy of choice for what we refer to here as the 'Yule-Lotka-Rosen-Adler' research stream.

First, but not actually part of the chain, Cook (1989) examines how well the distribution of US Top 40 hits among artists from the 1955-1984 period fits Lotka's 'law' and does not find it to be statistically significant. Nevertheless, he considers the deviation marginal and posits that it may simply stem from data contamination from *Billboard*'s known manipulations. Cook cites Price – and is even among the few to cite Hesbacher et al. – but does not relate his work to Rosen and Adler's theories.

The seminal papers in the Yule-Lotka-Rosen-Adler stream are inarguably Hamlen (1991), and Chung and Cox (1994). The former attempts to test Rosen's model

and, thus, must find a way to operationalize 'talent', which is obviously difficult. Hamlen solves this problem by measuring 'voice quality' using the harmonic content of how the artists sing the word 'love' on record. The expediency of this approach has been questioned by, among others, Schulze (2011), who submits Bob Dylan, Britney Spears, The Spice Girls, and AC/DC as examples where 'talent' might not be well-represented by the artist's voice quality. In a similar endeavour, Krueger (2005) applies the number of millimetres of print devoted to each artist in *The Rolling Stone Encyclopedia of Rock & Roll* as a scale for 'star quality'. Unfortunately, finding an objective, empirical measure of talent might be impossible, and we will not delve further into this part of the stream here. However, it is pertinent to point out that the emerging field referred to as 'hit song science' has commonalities with Hamlen's approach, but the overall objective is that of *predicting* success, usually based on the intrinsic characteristics of the songs, such as lyrics and audio features, but also by mining 'extrinsic' data, e.g. from social media platforms (Pachet, 2011). Equipped with the advantages that a laboratory-like setting offers, experimental researchers have apparently found ways to circumvent the 'talent problem' by obtaining a 'natural measure' of a song's quality by observing its popularity among persons that have not had the opportunity to have their preferences influenced by others (Salganik, Dodds, & Watts, 2006, p. 854).

More relevant here is the line from Chung and Cox (1994), who emphasize the 'close proximity between the assumptions underlying the Yule distribution and the superstar model proposed by Adler' (p. 772). The Yule distribution was given its name by Simon (1955) and is the limiting distribution of Yule's stochastic 'urn scheme' process introduced above:

[Equation 1 near here]

where p_k is the probability of measuring the value k and B(k, α) is the *Legendre beta function* (the Γ denotes the *gamma function*). Also known as the Yule-Simon, or even Simon-Yule, the Yule distribution is convenient because sums involving it are often tractable and can be solved in closed form.

Chung and Cox apply the number of gold records (for both albums and singles) awarded the by Recording Industry Association of America (RIAA) for the 1958-1989

period as a measure of success and find Yule to be 'an excellent abstraction' of how these records are distributed among different artists. Table 1 presents a summary of notable works following Chung and Cox. At the grave risk of oversimplifying, these contributions are made by statistically well-informed economists debating how well two datasets of #1 hits and gold records conform to Yule's theoretical distribution. Apparently, successive contributions have contested Chung and Cox's findings.

[Table 1 near here]

Under the illustrative title 'Superstars without Talent? The Yule Distribution Controversy', Spierdijk and Voorneveld (2009) set out to resolve the matter and apply no less than seven different tests to assess statistical validity. Yule is 'overwhelmingly rejected', as the authors find that the distribution captures stardom but not superstardom – in other words, the Yule distribution 'overestimates the snowball effect that makes consumers purchase records by the most successful artists' (p. 9). Anecdotally, both Fox and Kochanowski (2004) and Spierdijk and Voorneveld (2009) find the *generalized* Yule, a two-parameter variant that involves the *incomplete beta function*, to be 'an excellent fit'.

Spierdijk and Voorneveld inarguably provide valuable contributions to the discussion on the appropriateness of the analysis methodology; however, we believe the entire 'controversy' might be founded on inadequate data. In the real world of commercial music, the 'basic unit' is not gold or #1 records, but rather – and of course – money. We believe it likely that such crude and imperfect measures might fail as 'proxies' for the actual underlying economic distribution. To give just one example of its possible defectiveness: for every gold or #1 record that a typical superstar earns, several other *almost* equally successful records will typically follow in its wake. To illustrate, most readers should be familiar with the six consecutive classic Madonna singles released during the 1986-1987 period that are presented in Table 2: 'Live to Tell', 'Papa Don't Preach', 'True Blue', 'Open Your Heart', 'La Isla Bonita', and 'Who's That Girl'. All but one of these songs would not be included in either of the two datasets mentioned above because it had not been #1 or awarded a gold record, or, in the case of 'La Isla Bonita', neither. Moreover, 'I'm Yours' by Jason Mraz was unquestionably a monster hit and spent 76 weeks on the Hot 100 in 2008 and 2009; nevertheless, it never reached higher than #6. It is, however, RIAA certified seven times

platinum, an accomplishment very rare even among #1 records. Thus, we assert that is seems somewhat risky to assume that a simple #1 or gold record count will truthfully represent the economic realities; the appropriateness of this approach have never been justified. Hence, it may be that the form of measurement – not Yule's model – is not fit to 'capture' superstardom.

[Table 2 near here]

Notably, Giles (2006) calls attention to the 'sensitivity of the conclusions to the choice of "stardom measure" and concludes that 'much remains to be done, especially with respect to measuring success' (p. 73). We make the case here that *market share* is the most relevant measure of success when studying the music industry – including on the level of individual artists and songs. The multinational record company oligopoly itself (for the time being: Universal, Sony, and Warner) has long employed a market share approach and competes for market share, not gold records. The typical record company executive will most likely be willing to sacrifice profits to gain or maintain market share, as market share is the measure by which his own performance will be appraised (Negus, 1999, p. 45).

In the period we examine (1960-2010), the record industry has predominantly been a traditional transaction economy in which vinyl records, cassettes, CDs, and mp3s, among others, have been sold for a nonrecurring one-time charge. We are currently moving into what appears to be an age of *streaming* in which consumers simply pay for access to an entire music catalogue via a monthly subscription fee and the revenue pool is then distributed to the various rights holders based on their market share within the service. In this setting, record companies and artists now compete solely for market share – both implicitly and explicitly.

As a measure, market share has several qualities that recommend it. In contrast to gold or #1 records, market share can describe every type of chart 'success' – including very modest success. In addition, it is pertinent when comparing across different periods, as specific sales figures can vary over time due to numerous factors. Moreover, market shares can be meaningfully added together, e.g. the sum of a series of weekly attainments represents a value that is directly and quantitatively comparable to

any other such sum. Of course, this is what Hesbacher et al. realized, and the 'weights' they assigned to the different chart positions are essentially the same as market shares.

On a final note, Spierdijk and Voorneveld (2009) point out that the analytic procedure that Chung and Cox (1994) and others perform is in fact not a test of the Yule distribution but rather a test of a so-called *power law*, which they claim is merely an approximation. Hence, Chung and Cox found that the latter fits their data.

2.4 Power laws and market shares

In 1913, a German physicist named Felix Auerbach made a rather uncanny discovery: some J-curves, when plotted on a logarithmic axis, take the form of approximately straight lines (Rybski, 2013). Simply, J-curves that form straight logarithmic lines are said to adhere to a power law and can be expressed in form:

[Equation 2 near here]

where α is the *exponent* and C is the *normalizing constant*. In empirical distributions, power law behaviour is usually seen only in the tail of the distribution, for values $x \geq x_{min}$, but theoretically a power law can hold all the way down to $x = 1$ (Newman, 2005).

As a type of distribution, power laws have properties that are exclusive to them, such as 'scalability'; subordinate values do not have a characteristic scale; a power law is the same whatever scale we look at it on, or 'self-similarity'; cf. 'fractals', the big, aggregated patterns that are an assembly of many smaller similar patterns, e.g. 'Sierpinski triangle' and 'Koch snowflake' (see Mandelbrot (1983)). We believe it is noteworthy here to compare power laws to the more famous and familiar normal distribution, a.k.a. the Gaussian distribution. The latter frequently appears in settings in which events are random and independent, e.g. coin tossing, but also shows up in nature, such as in the distribution of height among humans. Although Gaussian distributions are quite orderly and can be described with just two numbers (the mean and the variance), power law distributions are more unruly; they have infinite variance, and some even have infinite mean values. As opposed to normal distributions, power law distributions are not dominated by the mediocre majority, but by the few extreme values – in our case, the 'superstar' songs and artists – and you never really know what

type of monsters might show up. Therefore, drawing a sample from a population while assuming the values in question are normally distributed, when the values are in fact distributed under a power law, can lead to severe selection bias. For instance, if you randomly choose one hundred female pop performers, the mean value of the number of their #1 hits will be extremely influenced by whether Madonna happens to be included or not – she will very likely have more #1s than the rest of the sample put together.

As already noted, power laws and Yule are closely connected. Generically, the Yule process yields a Yule distribution, which is just a beta function (cf. Eq. (1)). The beta function has a power law tail, but 'rolls off' at low values. In other words, the Yule process – i.e. in the form of the Yule-Simon urn scheme described above – generates a power law distribution. Because mechanisms other than cumulative advantage can also do this, power law distributions show up in an even more diverse range of areas than Yule distributions, but some relevant examples with moderate-to-good support include city sizes, the occurrence of words in English text, and the sales of music recordings (Newman, 2005).

2.4.1 'Appropriate proportion of designated popularity'

Hesbacher et al. (1982) propose an equation to 'satisfactorily measure each position's appropriate proportion of designated popularity', resulting in a shape said to be 'resembling the mirror image of a J-curve' (p. 101). The formula is incorrectly transcribed in the original article, a corrected version is provided by Carroll (2015, p. 595):

[Equation 3 near here]

where x is the rank on the chart, a value between 1 and 100 (both inclusive), and y is the 'weight', i.e. 'appropriate proportion of designated popularity'. After reviewing a range of methods, Carroll settles for an expression on the same form as Hesbacher et al., where y is altered to 'score' (2015, p. 597):

[Equation 4 near here]

Given Hesbacher et al.'s privileged access to Billboard's data and methodology, there is reason to suspect that their equation truthfully reflects the underlying realities in terms of record sales and radio exposure figures. However, we believe there is room for

further development of this approach. First, equations (3) and (4) formally only applies to the periods 1970–1979 and 1958-1975, respectively. As the number of annual chart entrants has been in steady decline since 1967 (see Figure 3), it does not seem safe to assume that the weekly 'popularity distribution' has remained unchanged during the 80s, 90s, and 00s, so a comparison against more recent data appears timely. Second, although being analogous concepts, we argue that 'market share' as a success measure is more comprehensible and meaningful in the current setting than 'weight' or 'score'. Third, as the current consensus is that recorded music sales follow a power law distribution, it seems reasonable to suggest that this is the appropriate form to employ. Evidently, sales figures and market shares are closely related, and it has been demonstrated that the pattern of market shares in product categories like foods and sporting goods are represented well by power laws (Kohli & Sah, 2006). Thus, based on the approach of Kohli and Sah (2006), we suggest the following expression to describe relationships between rank and market share in sound recording popularity charts:

[Equation 5 near here]

where C and α are constants, r is the rank on the chart, a value between 1 and 100 (both inclusive), and s is the 'market share', i.e. the share of the weekly total market value of all the top 100 records.

2.4.2 A Hot 100 power law formula: a first pass

For our current purposes, equation parameters suitable to represent the Billboard Hot 100 over a fifty year period from 1960-2010 need to be determined. Regrettably, unlike Hesbacher et al., we do not have wide access to 'field data', so their results will serve as a baseline and benchmark. For a more up-to-date account, a set of power law equation parameters are estimated empirically based on a sample of 12 randomly selected Digital Songs charts from the period between week 48, 2012 and week 32, 2013. The Digital Songs chart, compiled by Nielsen SoundScan[©], tracks the sales of the most popular songs in the U.S. and is an underlying component of the *Billboard* Hot 100.

The *max* value in the dataset, the highest weekly sales figure observed, belongs to the song 'Thrift Shop' by Macklemore & Ryan Lewis, with 412,336 registered sales in week 7, 2013 – a 5.94 % share of the combined volume of all the top 100 tracks that week (6,939,292). The *min* value is 16,885 from John Legend's 'All of Me' at #100 in

week 32, 2013. Notably, in week 33, 2012, the Taylor Swift song 'We Are Never Ever Getting Back Together' debuted at #1 in the Digital Songs chart with a weekly sale of 623,000 (Caulfield, 2012). Somewhat intriguingly, the following week the song still held the top spot, although its count had dropped by more than half to 307,000. Such fluctuations in the 'left-hand tail' obviously pose a modelling challenge. However, converting the data points to a weekly marked share percentage format (simply, for each weekly chart, each of the 100 individual sales figures are divided by the total) sometimes alters the scheme slightly. While 'Thrift Shop' had its highest sales in week 7, 2013, it actually had a higher market share in week 5, 2013 (7.6% − 381,056 out of a total volume of 5,012,673). Admittedly, this dataset is rather small and is even sampled outside the period under investigation (1960-2010). Moreover, sales is only one of the Hot 100 'constituents', as *Billboard*'s editorial director wrote in March 2013: "Generally speaking, our Hot 100 formula targets a ratio of sales (35-45%), airplay (30-40%) and streaming (20-30%)" (Werde, 2013, p. 7).

Apparently, identifying and measuring power laws in empirical data is a more complex task than most researchers realize (Clauset, Shalizi, & Newman, 2009); as a result, most reported power laws lack both sufficient statistical support and mechanistic backing (Stumpf & Porter, 2012). Fortunately, we seek merely a satisfactory approximation and leave to future research to characterize a Hot 100 power law more correctly. Hence, we here opt to estimate the power law parameters by the simple procedure of fitting an *ordinary least squares* (OLS) regression line to the empirical Auerbach line that is derived from the average market share for each chart position over the twelve weeks in the dataset (Figure 1). Conceptually, power law distributions and market shares have the common requirement of summing to 1. For a power law distribution, once the exponent α is fixed, the normalizing constant C is determined by the sum-to-1 requirement. Thus, C will be the market share assigned to the #1 chart position.

[Figure 1 near here]

This yields the following power law function for converting a weekly Digital Songs chart position into a marked share value:

[Equation 6 near here]

where s is market share and r is chart position. Figure 1 might give some cause for concern; the formula (6) appears to overestimate the market share held by the most popular songs. Note that the objective here is to approximate the Hot 100, which during the 1960-2010 period has always included airplay as well. The output from Hesbacher et al.'s (1982) and Carroll's (2015) formulas are easily numerically converted to a market share framework (for each chart positions x_i, the 'weight' is divided by the sum of 'weight' over all values of x), and Figure 2 shows how the resulting curves compare to our version. According to Carroll, the 'fat tail' of Hesbacher et al.'s distribution is 'driven by the steepness of the radio rating system' (2015, p. 595), implying that the distribution of airplay among popular songs is even more skewed than that of sales. We therefore assume that our equation can be accepted as a satisfactory approximation of the distributions of market share on the Hot 100 for the period in question.

[Figure 2 near here]

Thus, we now have a variable that can be considered a ratio scale, with all the convenience that such a scale offers. By adding together the appropriate series of weekly marked shares, we can now e.g. calculate that two weeks at #3 has approximately similar value to that of spending one week at #1, which again is approximately equally valuable to 20 weeks at #100. This approach will also manage to fully account for the success of a song like 'I'm Yours', by Jason Mraz and it will also assign it a value that faithfully represents what a big record it was.

3. Descriptive summary: The Hot 100

This section provides an example that demonstrates how a rank-to-market share equation like (6) can be utilized in combination with freely available chart data. The intent is also to make visible how this approach may yield a more comprehensive and detailed account of how success is distributed in the pop music economy. The analysis that follows is claimed to be no more than a descriptive summary, and the data are arranged for the purpose of rudimentary reflection over how historical Hot 100 chart data seem to align with theories of cumulative advantage.

3.1 Arranging the data: Billboard Hot 100 (1960-2010)

The data originate from a book series called *Joel Whitburn Presents the Billboard Hot 100 Chart*; each volume contains every weekly Hot 100 chart in a given ten-year period, e.g. the 1990s (Whitburn, 2000). Arguably, the weekly Hot 100 chart, which is a sequence of data points measured at successive points spaced at uniform time intervals, somewhat meets the criteria for a time-series of a cohort or panel but with different numbers of cases being replaced every week. However, for the current study, this way to arrange the data is not considered useful.

The data arrangement described below is based on the following reasoning: generally, a record is only released to the market once. If a song fails to attract interest from the market itself and/or from the media, the release process is unlikely to be repeated at a later stage; instead, the record will most likely be 'dropped' and forgotten promptly. This 'one-shot-only' feature allows for the possibility to treat all songs as existing in the same 'time and space'. All songs in our dataset were subject to the same conditions, regardless of era and environment; only 100 possible spots are available in any given week for any given song. The registration process begins when a song first enters the chart (e.g. '1st week: #64'), and some songs may drop out and re-enter at a later stage. Arranging the data in this fashion transforms the data into a longitudinal form that lends itself to more straightforward and simple methods of analysis (see Table 3).

[Table 3 near here]

3.2 A few descriptive figures

The dataset contains the complete chart history of 23,905 individual releases that have spent at least one week on the Hot 100. The songs are sorted reverse chronologically from 1960 to 2010 based on the year they reached their highest position and the date they first entered the chart. Therefore, 'Welcome Christmas' performed by the Glee Cast (December 25, 2010) is the first case, and the last is Johnny Preston's 'Running Bear' (October 12, 1959) – which entered the chart in 1959 but peaked in 1960. There are a few examples in which the same recording has been released more than once, and in these rare occurrences, a re-release is treated as a new, separate record. As shown in Figure 3, charted songs are not particularly evenly distributed across the years, clearly indicating how the chart 'dynamics' have changed – which is what Carroll's methodology intended to mitigate. The publishers of *Billboard* also compensate for these variations when they compile 'all-time' lists, such as *Hot 100 55th Anniversary: The All-Time Top 100 Songs*: 'Due to changes in chart methodology over the Hot 100's 55 years …, certain eras are weighted differently to account for chart turnover rates over various periods' (Bronson, 2013). However, in the current undertaking, we treat all songs on the same basis.

[Figure 3 near here]

'Although every song's path may be unique', Bradlow and Fader posit that a records lifespan on the Hot 100 can be described as a 'birth-growth-decline-death' process (2001, p. 369). Hence, the number of songs with a registered chart position declines as the week number increases (see Figure 6, right axis). As Figure 4 illustrates, the distribution of weeks spent in the chart over the 23,905 records yields a right-skewed curve centred at 7 weeks, but a more striking feature is the significant spike at week 20. This anomaly is also identified by Bradlow and Fader (2001) and is very likely the result of a manipulation referred to as 'early deletion' (see e.g. Cook (1989, p. 282)). By the 11th week, the number of songs still in the chart is reduced by more than half to 11,171. By week 26, it drops to triple digits (839 songs), and in the 43rd week, only 93 releases remain. Within the 1960-2010 period, only two songs are registered in the 76th week; 'Macarena' by Los Del Rio (1996) and 'I'm Yours' by Jason Mraz (2009). The dataset includes 6,384 recognized unique artists of which a half (49.66%)

had a song enter the Hot 100 only once. The number of individual data points in the form of registered chart positions for all songs sum to 267,738.

[Figure 4 near here]

3.3 Preliminary data processing

With reference to the data structure example provided in Table 3, Table 4(a) shows three arbitrary cases and their Hot 100 chart history. Then, a weekly chart position of, e.g. #59, will thus be converted to a market share value using the equation (6) introduced above, and the value 0 is assigned to all blank cells. For each case, the aggregated market share value over all weeks is then calculated in the variable **Total MS**. Table 4(b) shows the resulting chart.

[Table 4a near here]

[Table 4b near here]

Because the dataset is ordered in reverse chronology, the number of previously charted songs by the artist for each case are counted and entered into the **Preceding** variable. Similarly, the combined **Total MS** of the preceding songs is recorded in **Accu MS**. As an illustrative example of this arrangement, Table 5 shows the resulting dataset, which is limited to Madonna's career (note: the list is abbreviated, the complete version contains 54 songs). Notably, our methodology recognizes the song 'Take a Bow' as Madonna's all-time biggest hit, with a **Total MS** value of ≈1.09. As an expression of 'stardom', by the time the song 'Celebration' entered the Hot 100 on August 22, 2009, Madonna had already amassed an **Accu MS** value of ≈15.61. Likewise, her Hot 100 debut 'Holiday', which entered the chart on October 29, 1983, has an associated **Accu MS** value of exactly 0.

[Table 5 near here]

On a final note, the applied procedure relates only to artist concepts, and therefore, some individuals will appear in several different instances. For instance, Sean John Combs has released singles under various names since 1997, i.e. Puff Daddy, P. Diddy, and Diddy, and Diddy-Dirty Money, all of which will be treated as separate entities, following Chung and Cox (1994). The same applies for simple inconsistencies

in spelling, e.g. a band from Michigan appears as three different entities: '? & the Mysterians', '? and the Mysterians', and '? (Question Mark) and the Mysterians'.

We assume here that the data – the entire history of Hot 100 records for a specified period – represent a population; they are not a sample. This approach conveniently dodges several methodological issues and simplifies matters, but it will also affect the validity of the results. Nonetheless, the population should be unequivocal and well defined, as a song has either been on the Hot 100 or not. The dataset contains the complete chart history of all members of the population – songs that reached its highest position within the period 1960-2010 – so data from 1959 and 2011 are included where appropriate. Table 6 provides a summary of the key variables used in the analysis.

What follows should therefore not be taken as a test for the significance of the null hypothesis or something similar; instead, we simply make a descriptive summary of historical data. Hence, we make no formal attempts at casual inference; our objective here is just to describe the association between variables. We measure this relationship by Pearson correlation and, for good measure, we also report the results from the linear regression where appropriate. Pearson's r ranges between ± 1 (a +1 indicates a perfect positive correlation), like most correlation coefficients, and thus summarizes the relationship between two variables with a single number.

[Table 6 near here]

3.4 Methodological considerations

Pearson's r is a measure of linear association; in observance of the J-curve, our data appear to have certain non-linear properties. Unfortunately, measuring correlation in 'power law' data is tricky. Pearson's r requires both the mean and the standard deviation of the distributions, whereas a theoretical power law distribution does not necessarily provide such concepts. Naturally, the actual dataset offers numerically calculable mean and variance – and the *min* and *max* values are known. Moreover, this is certainly not an attempt to contribute to the science of measuring cumulative advantage; to be sure, there are already considerably more elegant approaches (e.g. Jeong, Néda, and Barabási, (2003)). Therefore, it is not particularly easy to extract

statistical information from 'fat-tailed' data; however, in this case, the data consist of the complete chronicles of a population for a specified period – and the dataset 'is what it is'.

We nevertheless attempt to summarize non-linear data in a linear framework. The coefficient of non-linear correlation – eta (η) – will be provided where it is deemed appropriate. The eta is the ratio of the between sum of squares to total sum of squares in analysis of variance (ANOVA) and is calculated by dividing one of the variables into groups of equal width according to their rank (Kennedy, 1970). The extent to which η is greater than r is here interpreted as an estimate of the extent to which the data relationship is non-linear.

3.5 A simple operationalization of cumulative advantage

We here recall the first 'main connotation' of DiPrete and Eirich (2006), as stated above, and create the following testable statement based on it:

1. The market share obtained by a record in the current week is positively associated with the market share obtained in the following week (*Snowball*)

We add another statement of similar form but shift the focus to the artist level:

2. The market share obtained by a current record is positively associated with the market share accumulated by the recording artist's earlier releases (*Star Power*)

For illustrative purposes only – and in their loosest meaning – we offer the following nicknames for these relationships; respectively *Snowball* and *Star Power*. Thus, the song-level 'snowball' cumulative advantage is assessed mainly by observing the separate levels of association between different measurement points in time, in other words, by simply contemplating the correlation matrix of all the weekly market shares. This approach remains wholly descriptive; we merely summarize 'the spectrum of the process' – whatever it is – and make no assumptions regarding any particular structure.

Based on similar reasoning, the **Accu MS** value represents the artist's cumulative advantage – her 'favourable relative position' or her 'star power' – at the time the song with the corresponding **Total MS** value entered the chart. If this general

and inherent 'influence' on events exists – regardless of the specific mechanisms involved – there must be a positive association between these two variables.

4. Results and discussion

The correlation analysis is implemented for a total of 47 relevant variables and in a single matrix; therefore, all the results reported below originate from this one table. Because a 47*47 matrix does not lend itself to meaningful reproduction in the current format, it is not included here. An HTML-formatted version of the matrix is included in the supplemental material and can thus be explored in a more user-friendly manner with a standard web browser. However, in the current setting, the 'shape' of the correlation matrix itself might be interesting in its own right. Because it is frequently easier to obtain an impression of such a compilation of figures when presented graphically, a so-called contour plot of the matrix is also provided (Figure 5). At the risk of being repetitive, because we claim to be considering a population, the significance level is not necessary of particular interest; nonetheless, all the coefficients reported are significant at the 0.01 level (2-tailed).

[Figure 5 near here]

4.1 Snowball

In relation to the first statement formulated above, perhaps the most striking and telling descriptive representation of the data can be found in Figure 6, left axis. The graph illustrate an excerpt from the matrix, i.e. the diagonal of the pairwise, week-on-week correlation between consecutive weeks, otherwise known as autocorrelation with lag of 1.

[Figure 6 near here]

For example, the week-on-week associations in market share, i.e. the correlation between the **2nd Week** and the **1st Week** is $r = 0.717$; for the **5th Week** and the **4th Week**, it is $r = 0.914$; and so on. Summarized in the form of linear regression, an r here of 0.914 amounts to an *adjusted r squared* (a.k.a. the coefficient of determination) of 0.835. However, the objective of this undertaking is not, for example, to estimate the parameters of a model describing a stochastic cumulative advantage process or something similar. For a descriptive approach, the contour plot representation of the correlation matrix found in Figure 5 provides a simple illustration of the 'spectrum of

the process'. It bears unequivocal testimony to 'seriality' being present; in fact, even the very extremities appear to be associated. Figure 7 contains four 'arbitrary' examples of similar week-on-week associations in the form of scatter plots. Animation 1(a) (provided in the supplemental material) demonstrates how this scatter plot pattern evolves from week to week.

[Figure 7 near here]

To an observer that merely studies sales and airplay figures (and perhaps disregards the first four weeks) for the 1960-2010 period, it might certainly appear that there was a phenomenon taking place that would be adequately described with a snowball metaphor, i.e. a Yule process of proportional growth. Arguably, it should not require any mathematical formality to account for the fact that in an environment in which next week's outcome for any participant is generally ($r > 0.9$) associated with that of the current week, the larger snowball will continue to draw proportionally more snow, and the end result of this process should be easy to envision. Simplified, an artist selling 300,000 copies will continue selling an approximately similar amount – in the same order of magnitude – the following period; unfortunately, the same applies for a participant selling just 10,000 copies. The relative ratios will remain intact, so after ten weeks, for example, the greater-selling participant has snowballed into a 'star', whereas the lesser-selling participant has merely found its place somewhere among the numerous inhabitants of the 'long tail'. Each week, a number of songs lose momentum, stop 'rolling', and fall off the chart; the majority of these songs will be of the smaller snowballs (they form the small 'pillar' to the very left in the scatter plots in Figure 7). However, it is important to remember that the J-curve is in itself not a result of a longer-term process; a similar shape also appears on a weekly basis. This might also provide a notion of the fractal nature of the power law phenomenon: the aggregated pattern is made up of many smaller similar patterns – i.e. the 'self-similarity' property mentioned earlier.

4.1.1 The first four weeks

The depictions in Figures 6 and 7 pose another question that goes to the heart of the Rosen/Adler dichotomy: *what is going on in the first four weeks?* The scatter plots in Figure 7 show the week-on-week market share comparison at four different points in

time and arguably portray some type of arrangement that appears to begin rather chaotically but that quickly takes on a distinct shape that again seems to become increasingly significant over time (see also Animation 1(a), and 1(b)). In other words, the Hot 100 market appears to have generally needed four weeks to 'get a grip on' a new song, and from then on, that song's destiny is largely determined.

Are we just seeing the footprints of a game of chance sorting itself out? The market may have been collectively undertaking a complex four-week quality assessment routine, or the scatter plots may have shown the impression of a random social process that would have occurred in this way regardless of how the songs were introduced? These questions are not exclusive to our setting but instead belong to a slowly evolving debate that has gone on for more than 50 years: *is preferential attachment rooted in pure chance or in some form of optimization?* (Barabási, 2012). To be sure, authors other than Rosen and Adler have found themselves on opposite sides of this issue. In the 1960s, the aforementioned Herbert A. Simon and Benoît Mandelbrot engaged in a fierce public dispute, with the former arguing that random preferential attachment explains the power law distribution of word frequencies in text, and the latter arguing that it is the result of optimization (Perc, 2014, p. 10) – i.e. that language is being developed to transmit information most efficiently. Since the turn of the millennium, experimental evidence supporting preferential attachment in the context of networks has arguably accumulated in support of the former view, but the debate remains open. In the words of Barabási (2012):

> The fact that the effect is widespread suggests that it probably derives from
>
> both agency and random actions. Most complex systems have a bit of both,
>
> so we do not need to choose between them. Luck or reason, preferential
>
> attachment wins either way. And so do we, gaining a deeper understanding
>
> of this puzzling yet ubiquitous force. (p. 507)

This conclusion will perhaps not be considered sensational by most observers of the market for pop songs. Even before the term 'rock and roll' was coined, Leibenstein (1950) included both 'functional' and 'non-functional' elements (of which the 'bandwagon effect' was an 'external effect on utility') in his theoretical model of

consumer demand motivation. Hesbacher et al.'s (1975b) model 'factors influencing sound recording popularity' also included both intrinsic and extrinsic factors. This 'a bit of both' notion also corresponds with results from large-scale experiments that found that 'success was also only partly determined by quality: The best songs rarely did poorly, and the worst rarely did well, but any other result was possible' (Salganik et al., 2006, p. 854).

4.2 Star Power

[Table 7 near here]

Table 7 presents the main results from the analysis with respect to statement (2) above. The variables **Accu MS(2000)**, and **Accu MS(500)** are similar to **Accu MS**, but with shorter 'memory'; they only 'remember' the 2,000 and 500 previously charted songs. For example, **Accu MS(2000)** indicates the value for a given song in the respective recording artist's accumulated market share by preceding singles among the 2,000 previous cases (songs).

Appraised in a linear framework, recent success seems to have a higher and longer-lasting association with current success than older success. We can also observe how this 'star power' association varies over time. Figure 8 depicts how the three **Accu MS** variables correlate with the different individual weekly outcomes. Arguably, the following quote from Bradlow and Fader provides an apt description of the graphs, i.e. 'historically popular artists find it easier to move higher in the chart more quickly, remain up high for longer, and have an overall longer stay' (2001, p. 369).

[Figure 8 near here]

If we also further summarize the data here with linear regression, the calculation with respect to **Accu MS** yields an adjusted r^2 of 0.014 (if the cases without previous chart history are removed from the analysis, we obtain a marginally better result of $r = 0.015$ and adjusted $r^2 = 0.022$). The similar figure for **Accu MS(2000)** is more than double the previous figure: 3.2%. Regardless, this amount might not sound like much. At first glance, star power might seem negligible – at least in this manner of operationalization and appraisal. Presumably, star power is of little significance on the

individual artist level – a notion supported by the low rate of repeat visitors. In a somewhat similar assessment of the movie industry, De Vany and Walls concluded the following: 'the audience makes a movie a hit and no amount of "star power" or marketing can alter that. In other words, the real star is the movie' (1999, p. 285). The appreciation of these percentage figures might depend on one's perspective and perception of the 'game'. A Las Vegas blackjack gambler would definitely be happy with an advantage of similar magnitude. In fact, a typical 'house edge' – the built-in advantage a casino has on its blackjack games – is ~0.5% (see e.g. 'Blackjack House Edge - Wizard of Odds', n.d). That does not sound like much, but it is evidently enough to 'tilt the table' sufficiently in favour of the house to allow a lucrative business.

Similarly, for the small group of oligopolists that has dominated the recording industry for more or less its entire existence (Belinfante & Johnson, 1982), 'star power', even just in the form of a correlation-based prediction, may actually provide a small but vital edge that 'tilts the table' sufficiently in their favour. Because *nobody knows*; the demand for new songs is highly uncertain, and because historical estimates suggest that 85% of the single records released throughout history have not even recouped their associated expenses (Caves, 2000), this setting is perhaps not so different from that of a casino. Moreover, because it is costly to retain and cultivate superstars, this phenomenon will also act as a 'barrier to entry' keeping smaller record companies out of the market. Over the years, the oligopoly have apparently become increasingly rooted in what Elberse calls 'blockbuster strategies'; the competition is for the next superstar because that is where the crucial market shares are made or lost (2013, p. 250).

4.3 The case of Madonna

Finally, we return to Madonna for an anecdotal example. Figure 9 provides a scatter plot of the 54 Madonna songs in our dataset. Arguably, we can envision a trend line among the points sloping downwards from left to right, and this trend would imply that her star power association is in fact negative, a somewhat sensational and dramatic circumstance. Indeed, a statistical summary of Madonna's numbers yields a negative r of -0.3 ($p = 0.028$), which translates to an adjusted r^2 of 0.072.

[Figure 9 near here]

Presumably, it is well known that stardom is not a unidirectional and constant blessing – the 'favourable relative position' can turn into a curse. In a comment to Rosen's seminal article, Bowbrick (1983) quotes David Ogilvy, a.k.a. the Father of Advertising, who reveals a somewhat dark secret from the movie industry in the 1930s:

> I discovered that some stars had a negative effect at the box office; their names on the marquee repelled more ticket buyers than they attracted. The list, which I called Box Office Poison and classified TOP SECRET, included some of the most famous names in show business, and ruined their careers. (p. 459)

As for other factors with negative signs, Leibenstein's (1950) model mentioned above includes a 'snob effect' in which 'demand for a consumers' good is decreased owing to the fact that others are also consuming the same commodity (or that others are increasing their consumption of that commodity)' (p. 189).

[Figure 10a near here]

Consequently, it appears that cumulative advantage is not a constant, linear, and unidirectional effect and thus cannot be fully uncovered in a linear framework. The scatter plots in Figure 10(a) and 10(b) yield little in the way of discernible patterns (note: almost half of the songs are in the 'pillar' to the left – all the 'one hit wonders' in addition to all the other chart debuts like e.g. Madonna's 'Holiday'), and the eta (η) estimate (see Table 7) indicates that the relationship between the variables is largely non-linear.

4.4 Life and death

Notably, although the general week-on-week correlation is high – it is not a perfect 1. In a largely methodological paper using a dataset of 248 Hot 100 songs from 1993, Bradlow and Fader claim to identify a 'fairly complex "death process" that is rather hard to detect in simple summaries of the raw data' (2001, p. 378). They observe that songs fall very quickly after leaving the Top 40, and that a 20-week lifespan on the

chart seems to have particular hazardous qualities. Support for the latter claim can be found in our Figure 4; the most common fate appears to be elimination by 'early deletion' in the 20th week.

Although some artists and records can persevere for a considerable length of time – e.g. Pink Floyd's *The Dark Side of the Moon* spent 917 weeks on *Billboard*'s album chart between 1973 and 1988 (Caulfield, 2015) – no lifespan in the hit song industry seems to be eternal. Madonna may still be among the most recognizable individuals on the planet, but as a recording artist she has not had a #1 on the Hot 100 since year 2000. In fact, apart from the Super Bowl song, Madonna has only had one song on the Hot 100 in the 2010s ('Bitch I'm Madonna', two weeks in 2015: #84 and #95).

While a considerable body of literature has accumulated that encompass various approaches to uncovering the 'rules' that explains or describes how stars and hits are made (see e.g. Walls (2014) or Thompson (2017)), little attention have been paid to what one might, somewhat unsentimentally, call the 'product life cycle' of popular artist and songs. That is, the consideration of not only how they rise, but also how they fall. In addition to Bradlow and Fader (2001) – who introduce what they coin a Bayesian latent lifetime process, based on a stochastic utility model – a few attempts have been made by employing so-called duration models in the form of survival and hazard functions, e.g. Strobl and Tucker (2000), Bhattacharjee, Gopal, Lertwachara, Marsden, and Telang (2007), and Giles (2007). The 'survival' that is modelled is the duration a record stays on the charts, governed by a stochastic process with a one-week time index. Apparently, J-shaped curves show up in the resulting chart survival estimates (see e.g. Strobl and Tucker (2000, p. 128)). The hazard functions are similar but address the 'death', i.e. the event of exiting the chart.

In addition to the time factor, a small variety of explanatory variables and covariates have been included in these models, e.g. Bradlow and Fader (2001) settled for two measures: the artist's number of previous Hot 100 songs and a binary variable indicating whether the song appeared on a movie soundtrack. Bhattacharjee et al. (2007), who studied the album charts, added measures describing the record company (minor/major) and the artist (female/male/group). Arguably, the character of these works are predominantly methodological and descriptive, and it seems reasonable to

assume that the selection of variables is influenced more by (lack of) availability of data than of theory, but duration models appear as a promising approach for capturing the dynamics of the hit song system. The indications put forward in the current article do not contradict any of the results from these models, which, among other things, provide evidence for both a superstar and a snowball effect, and for the advantage of 'initial popularity' (a high chart debut) – all of which again align nicely with the connotations of cumulative advantage articulated by DiPrete and Eirich (2006). However, we dare suggest that future efforts in duration modelling may consider a marked share approach in place of the ordinal chart rank used in the existing literature – a modification that may facilitate the application of such models to artist careers as well.

[Figure 10b near here]

5. Conclusion

Confronted with highly esteemed champions of the Hot 100 'tournament', e.g. Elvis Presley (121 chart entries in our dataset), the Beatles (80), or Aretha Franklin (75), to mention just three, it is not tempting to fight the case for pure randomness. Even more astonishingly, the Beatles at one point managed to hold all five top spots on the Hot 100 simultaneously, truly a remarkable achievement (see *Billboard* April 4, 1964). The probability of such an outcome due to pure chance is truly infinitesimal and thus presents a challenge to the 'randomness' perspective and may even be interpreted as debunking the infamous *hot hand fallacy* (see Gilovich, Vallone, and Tversky (1985)).

Certain aspects of the pop music industry may deserve somewhat more emphasis in the superstar discussion than what it typically is awarded in the literature. The hit song economy must be considered peculiar as far as markets go; it has certain properties that are not found in many other industries, at least not to the same degree. We can argue that the main premise and objective of the pop music media industry is to elect 'the chosen few', to single out the officially most popular songs. It does not seem plausible that the influx of talent and quality will be perfectly constant over time; regardless, every week, the hundred spots of the Hot 100 must be filled by different songs, and one fortunate participant will find itself at the top. This ranking process must be performed for the songs available at the time, and there are no absolute requirements. In other words, the entire hit song 'food chain' can be said to exist for the primary purpose of creating hits; special songs will then receive equally special treatment and a lot more attention than the rest of the participants combined. Again, this market feature can be interpreted as support for the Rosen position that you do not have to be outstanding to bubble to the top of this hierarchy; you must only be a little bit better than the others. Unfortunately, these circumstances do not exclude the randomness argument: much like in a hat-drawing lottery, one song will *have* to be #1 every week – you must only happen to be the lucky winner. Our analysis, as vividly illustrated by the scatter plots depicted above, does not reveal whether this ranking process is deterministic, random or a combination – the same patterns might appear in all cases.

In conclusion, the current study emphatically suggests that cumulative advantage is a striking characteristic of the Hot 100 market, particularly on the level of individual songs. This process will again lead to 'path dependencies', e.g. it appears to

be difficult to sell many copies in the 11th week if you have not already sold a substantial number of copies in the preceding ten weeks. This path dependence effect is presumably so erratic that it sometimes accidentally propels poor-quality songs and untalented artists to the top of the charts, a phenomenon also demonstrated by experimental research (Salganik & Watts, 2008).

6. Future Research

The single was the core product in the recording industry for most of the 20th century, but was eventually eclipsed by the album format during the 1970s (Millard, 2005), and for several decades, albums have been the primary source of revenues for record companies. Now – in what appears to be a new paradigm in which streaming will come to dominate – it appears that the individual track will again become the bread and butter of the recording industry. Thus, the single deserves particular attention.

The relationship between talent and success in popular music might seem like a mystery beyond the reach of science. However, the prospects for gaining a deeper understanding of these connections have never been better. For example, the streaming service provider Spotify currently shares its top 200 chart in (daily) minute detail ('Spotify Charts', n.d.). Such rich and accurate data, combined with the swiftly advancing research field of music information retrieval (MIR), represents infinite opportunities for future endeavours into the economics of superstars.

As 'success measure', the current state of the art in the relatively new area referred to as 'hit song science' usually employ a binary 'hit/non-hit' variable based on certain cutoff chart positions (e.g. Herremans et al. (2014), Nunes and Ordanini (2014)). We believe that a market share approach can make such analyses more sensitive, with the inherent potential of uncovering more subtle relationships. The equation (6) provided in this paper is considered preliminary; however, advancement will require wider access to field data. Future research could also consider whether the Yule distribution might provide a better fit to such data than a power law.

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Table 1. Notable works from the Yule-Lotka-Rosen-Adler research stream.

Work	Theoretical framework	Success measure	Support (Lotka and/or Yule)
Cook (1989)	Lotka	Top 40 hits	No ('marginal deviation')
Chung and Cox (1994)	Yule, Rosen, Adler	Gold records	Yes ('excellent fit')
Cox, Felton, and Chung (1995)	Lotka	Gold records	No
Crain and Tollison (2002)	Rosen, Adler	#1 hits, weeks	N/A
Fox and Kochanowski (2004)	Lotka, Yule, Rosen, Adler	Gold records	No and No
Giles (2006)	Yule, Rosen, Adler	#1 hits, weeks	No ('overwhelmingly rejected')
Fox and Kochanowski (2007)	Yule, Rosen, Adler	Gold records	N/A
Spierdijk and Voorneveld (2009)	Yule, Rosen, Adler	All of the above	No ('overwhelmingly rejected')

Table 2. Six consecutive Madonna singles from 1986-1987.

Song	Hot 100 peak position	RIAA certification
Live to Tell	#1	
Papa Don't Preach	#1	Gold
True Blue	#3	Gold
Open Your Heart	#1	
La Isla Bonita	#4	
Who's That Girl	#1	

Table 3. Example of data structure (weekly chart positions).

Song	1st Week	2nd Week	…	75th Week	76th Week
Song 1	93	78	…	49	48
Song 2	5	22	…		
…	…	….	…	…	…
Song 23904	80	83	…		
Song 23905	75	64	…		

Table 4(a). Data example - weekly chart positions.

Artist	Track	1st Week	2nd Week	3rd Week	4th Week	5th Week	6th Week	...	76th Week
Glee Cast	Welcome Christmas	59						...	
Glee Cast	Baby, It's Cold Outside	57	99					...	
T.I.	That's All She Wrote	18	31	68	88	99		...	

Table 4(b). Data example - weekly market share values.

Artist	Track	1st Week	2nd Week	3rd Week	4th Week	5th Week	...	76th Week	Total MS
Glee Cast	Welcome Christmas	0.0061	0	0	0	0	...	0	0.0061
Glee Cast	Baby, It's Cold Outside	0.0062	0.0043	0	0	0	...	0	0.0106
T.I.	That's All She Wrote	0.0134	0.0093	0.0056	0.0047	0.0043	...	0	0.0373

Artist	Track	1st Week	2nd Week	3rd Week	4th Week	5th Week	...	76th Week	Total MS

Table 5. Example of data structure: Madonna Hot 100 songs (1983-2009), abbreviated.

Date entered	Track	# of weeks	Peak pos.	Total MS	Preceding	Accu MS
22 August 2009	Celebration	1	71	0.0054	53	15.6081
17 May 2008	Give It 2 Me	1	57	0.0062	52	15.6019
5 April 5 2008	4 Minutes	20	3	0.3924	51	15.2095
11 March 2006	Sorry	6	58	0.0314	50	15.1782
5 November 2005	Hung Up	20	7	0.2257	49	14.9524
5 April 5 2003	American Life	8	37	0.0460	48	14.9064
19 October 2002	Die Another Day	17	8	0.1992	47	14.7072
5 May 2001	What It Feels Like for a Girl	10	23	0.0783	46	14.6289
9 December 2000	Don't Tell Me	21	4	0.3586	45	14.2704
12 August 2000	Music	24	1	0.9026	44	13.3677
19 February 2000	American Pie	9	29	0.0725	43	13.2953
12 June 1999	Beautiful Stranger	19	19	0.1530	42	13.1423
…	…	…	…	…	…	…
17 December 1994	Take a Bow	30	1	1.0904	31	10.4162
…	…	…	…	…	…	…
18 March 1989	Like a Prayer	16	1	0.5052	15	5.3855
12 September 1987	Causing a Commotion	18	2	0.3405	14	5.0451
11 July 1987	Who's That Girl	16	1	0.3832	13	4.6619
21 March 1987	La Isla Bonita	17	4	0.2644	12	4.3975
6 December 1986	Open Your Heart	18	1	0.3972	11	4.0002
4 October 1986	True Blue	16	3	0.2951	10	3.7051
28 June 1986	Papa Don't Preach	18	1	0.4665	9	3.2386
12 April 1986	Live to Tell	18	1	0.4521	8	2.7866
17 August 1985	Dress You Up	16	5	0.2114	7	2.5752
27 April 1985	Angel	17	5	0.2294	6	2.3457
2 March 1985	Crazy for You	21	1	0.5474	5	1.7983
9 February 1985	Material Girl	17	2	0.3620	4	1.4364
17 November 1984	Like a Virgin	19	1	0.7221	3	0.7142
25 August 1984	Lucky Star	16	4	0.2547	2	0.4596
10 March 1984	Borderline	30	10	0.2833	1	0.1763
29 October 1983	Holiday	21	16	0.1763	0	0

Table 6. Summary of variables: Billboard Hot 100 dataset (1960-2010).

Variable	Description	Example value
Year	The year the recording attained its highest chart position	2009
Date entered	The date the recording entered the chart (i.e. 1st Week)	22 August 2009
Artist	Name of recording artist	Madonna
Track	Title of recording	Celebration
# of Weeks	The total number of weeks the recording spent in the chart	1
Peak Pos	The highest chart position attained by the recording	71
nth Week	The recording's chart position in week n (for n = 1, 2, 3,..., 76)	71
nth Week	The recording's market share in week n (for n = 1, 2, 3,..., 76)	0.0054
Total MS	The recording's total market share over all 76 weeks	0.0054
Preceding	The recording artist number of previously charted recordings	53
Accu MS	The recording artist's accumulated market share based on all successive cases in the dataset	15.6081
Accu MS(2000)	The recording artist's accumulated market share based on the next 2000 successive cases in the dataset	0.6557
Accu MS(500)	The recording artist's accumulated market share based on the next 500 successive cases in the dataset	0

Table 7. Main results from correlation and regression analysis: **Total MS – Accu MS**.

		Accu MS	Accu MS(2000)	Accu MS(500)
Total MS	N	23905	23905	23905
	Pearson correlation (r)	.118	.180	.176
	Adjusted r^2	.014	.032	.031
	Sig. (2-tailed)	.000	.000	.000
	Eta (η)	.858	.841	.761
	Eta squared (η^2)	.737	.708	.579

Fig. 1. Rank-to-market share equation: 'plain' OLS estimation of power law parameters. Data: Nielsen$^{©}$ Digital Songs, average market share for top 100 chart positions over 12 randomly selected weeks (2012–2013).

Fig. 2. Three J-curve equations compared: chart positions' designated market share of top 100 total.

Fig. 3. Billboard Hot 100: number of chart entries per year (1960–2010: 23,905 songs).

Fig. 4. Billboard Hot 100: distribution of songs over total number of weeks spent in the chart (1960–2010: 23,905 songs).

Fig. 5. Contour plot: correlation r matrix of weekly market shares (**1st Week: 26th Week**).

Fig. 6. Left axis: Correlation r between weekly market shares: n-th week and preceding week. Right axis: Number of songs with a chart position in n-th week.

Fig. 7. Scatter plots: weekly market shares, n-th week by following week (four 'arbitrary' examples).

Fig. 8. Correlation r between **Accu MS** and n-th week market share (**1st Week**: **26th Week**).

Fig. 9. Scatter plot: **Total MS** by **Accu MS,** Madonna's 54 songs.

Fig. 10(a). Scatter plot: **Total MS** by **Accu MS**, all 23,905 songs.

Fig. 10(b). Scatter plot: **Total MS** by **Accu MS**, zoom bottom-left corner.