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OPTIMAL ROTATIONS WITH DECLINING DISCOUNT RATE: INCORPORATING THINNING REVENUES AND CROP FORMATION COSTS IN A CROSS-EUROPEAN COMPARISON

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Abstract

Schedules of declining discount rates have been advocated, and adopted by several European governments. They undermine classical solutions to forest economics problems, especially optimal rotation. Adapting classical first-order conditions had created problems of local optimisation. A global search algorithm allowed inclusion of initial costs and thinning revenues. It produced results according with expectations – lengthening rotations as chronological time progressed and discount rate declined – and results paralleling those for constant discount rates – shorter rotations for high productivity and unthinned crops, and with zero crop formation costs. Apparent anomalies in the pattern of rotations are the explicable result from opportunity costs of delaying later rotations, which increase as discount rate declines. Sometimes the solution oscillates, usually owing to steps in the discount schedule or irregular profile of felling revenues. Inspection then allows the most profitable sequence of rotations to be identified.

Keywords: declining discount rate, optimal forest rotation, intermediate cash flows

Introduction

The discount rate is a key factor in forest economics (Johansson and Löfgren, 1985; Price, 1989). It strongly affects the optimal forest rotation (see Newman, 2002). Yet many academic arguments have been made, that discount rates should decline over time (e.g. Kula (1981); Price and Nair (1985); Weitzman (1998); Li and Löfgren (2000); Gollier (2002); Newell and Pizer, (2004)). These arguments are summarised in Price (1993) and Price (2011). They partly address sustainability concerns, and partly result from combining different discount rates for different products, different income trajectories or different future scenarios. Numerous psychological studies have also shown that different delays of gratification entail declining discount rates (e.g. Ainslie, 1991).

Now, several European governments have mandated use of such rates, as shown in table 1, for public sector appraisal: UK (UK Treasury, undated), France (Lebègue et al., 2005), Denmark (Finansministeriet, 2013) and Norway (Det Kongelige Finansdepartment, 2014)).

Table 1: UK, French, Danish and Norwegian discount rate schedules through future time

Period (years from present)	UK	France	Denmark	Norway
0-30	3.5%	4%	4%	4%
30-35	3%	2%	4%	4%
35-70	3%	2%	3%	3%
70-75	3%	2%	2%	3%
75-120	2.5%	2%	2%	2%
120-200	2%	2%	2%	2%
200-300	1.5%	2%	2%	2%
300-∞	1%	2%	2%	2%

Note: the figures are for discount rates used in quotient format, according to the formula [present value] = [cash flow at time t] / (1+[discount rate])^t. It is with these figures and in this format that the calculations in this paper are made.

A case can be made against this declining-discount approach to valuing the future (Price, 1993, 2004, 2005), and other means do exist to tackle the underlying problems (Price 2017a). However, if such rates *are* deployed, they undermine the basis of classical forest economics (Price, 2012).

This paper considers how these declining discount schedules affect the optimal sequence of forest rotations, and gives illustrative results for real forest cash flows in several European countries. It confines itself to the traditional elements of forest economics, silvicultural costs and timber revenues. Some comment is made in the Conclusions about how other costs and benefits might be included. It does not deal with the problem of dynamic inconsistency identified by Strotz (1956). This problem arises when the profile of discount rates shifts forwards through time, leading later decision makers to consider the optimal sequence of rotations as being different from the sequence imposed by an earlier decision maker. Price (2011) discusses the resulting inconsistencies. Instead, it takes the optimal sequence of rotations only as it appears from the perspective of a present-day decision maker. If the profile of discount rates is anchored to historical time, rather than shifting forwards as time elapses, such a perspective will determine an optimal sequence of rotations that should be consistently followed by future decision makers. Such historical anchoring is appropriate in such circumstances as are treated in Price (2017a).

Some of the results were presented at the IUFRO 125th Anniversary Meeting in Freiburg in 2017, and at the 2018 conference of the Scandinavian Society of Forest Economics.

The classical approach doesn’t work

The land expectation value (LEV) is the value of a series of perpetually recurring cash flows on a piece of land. Its calculation is problematical when discount rates vary through time. In particular, a central problem in forest economics, determining optimal forest rotation, can no longer be solved by applying the formula of Faustmann (1849), which has been extensively used for this purpose in both private and state sectors. Classically, according to the Faustmann formula,

[LEV of a perpetual series of rotations] =

[Net present value (NPV) of first rotation]/(1–$e^{-\rho T}$).

The problem arises because

- the discount rate, ρ, changes through historical time, so that
- the optimal rotation, T, also changes, lengthening as discount rate declines.

Hence

- the multiplier $(1/(1–e^{-\rho T}))$ from the NPV of the first crop to the LEV could contain no unique discount rate or rotation, and
- the first rotation's value, however calculated, would not be representative of the following crops' rotations, and
- the time lapse, T, between one crop and the next also varies.

Instead of a unique rotation that is optimal for each succeeding forest crop, there is an optimal *sequence* of differing rotations. This is unlikely to be determinable algebraically, particularly when the discount rate changes discontinuously, as it does in the schedules shown in table 1.

Brazee (2018) has derived algebraically the conditions for optimal harvest ages when discount rates are declining, but that approach has not so far been applied to numerical results for specific cases.

To avoid confusion in the following discussion, the term "rotation" will be used strictly to denote the length of time between crop initiation and crop termination. "Crop" will be used to denote one individual cycle of crop growth within the sequence of cycles.

The duration and value of each crop affects the position in chronological time of succeeding crops. This affects the schedule of discount rates applicable to them, and hence those crops' values and their own rotations' lengths. Moreover, LEVs of successor crops affect the opportunity cost of prolonging earlier rotations. Hence rotations cannot be optimised individually, without reference to other crops in the sequence. This creates a potential computational problem: even when the possibilities are limited to ten successive crops, each with rotation of anything up to 200 years, the number of permutations is 200^{10}: evaluating them sequentially would take thousands of years. Hence an intelligent search algorithm is needed.

A "global search" algorithm

For the smoothly varying cash flows produced by an algebraic revenue function, a backwards recursive protocol had yielded a stable and comprehensible solution (Price, 2011). Later rotations were, as expected, longer because of the discount rate's decline through time. A first-order condition identified individual crops' optima, when the increased benefit of bringing forward the cash flows of all successor crops equalled the benefit reduction from shortening the current crop's rotation. Because of the interaction between the rotations of each of the crops, an iterative algorithm was needed to determine the final optimum sequence.

However, problems arose when revenues from the generalised formula were replaced by real cash flow data, which included costs of crop formation and revenue from thinnings. The backwards recursive solution now produced both anomalous and unstable solutions. This was attributed to identification of local optima, especially in years when thinnings were made (Price et al., 2016).

The problems of the backwards recursive protocol were circumvented by a radically different, global search algorithm, less elegant than the first-order solutions previously adopted, but not susceptible to their problems of local optimisation and instability. It was presented at the IUFRO 125th Anniversary Meeting in Freiburg in 2017. Price et al. (2018) review the development of search algorithms more fully. The following sections enlarge on and expand the results from those presentations. The algorithm was applied to real-world cash flow and discounting schedules from UK, France, Norway, Denmark and Germany, allowing cross-European comparisons. The UK's discount schedule declines over the longest period, so this is used extensively in the following examples. The protocol was as follows.

- The starting conditions were provisional rotations of 200 years for each crop: in the event, no rotation as long as this was identified in any solution, and use of shorter provisional rotations over a wide range did not affect the solution.

- Crop formation costs and thinning revenues were included at their prescribed time within the crop's rotation. Rotation length determined magnitude of final felling revenues. All revenues were net of harvesting costs. Projected thinning and yearly felling revenues over a wide range of rotations were available for the UK from yield models and price data supplied by the UK Forestry Commission. Revenues were available at 5-year intervals for the other four countries. Danish data were compiled using material from Institut for Geovidenskab og Naturforvaltning, and Dansk Skovforeningen. German figures came from Schober (1995) and Sachsen Ministry of Environment and Agriculture (2000). Those for France were derived from the French National Forest Service. Norway's were based on Braastad (1975) and Vestjordet (1967). For these countries, yearly felling revenues were estimated by linear interpolation over 5-year periods. This facilitated optimisation to the nearest year, though the slight cusps at the 5-year points resulted in frequent results at those points. The revenues expected from final felling are plotted in figure 1, shown in euro equivalents (2018) for comparison purposes. Fuller details of cash flows for each country may be obtained from individual authors.

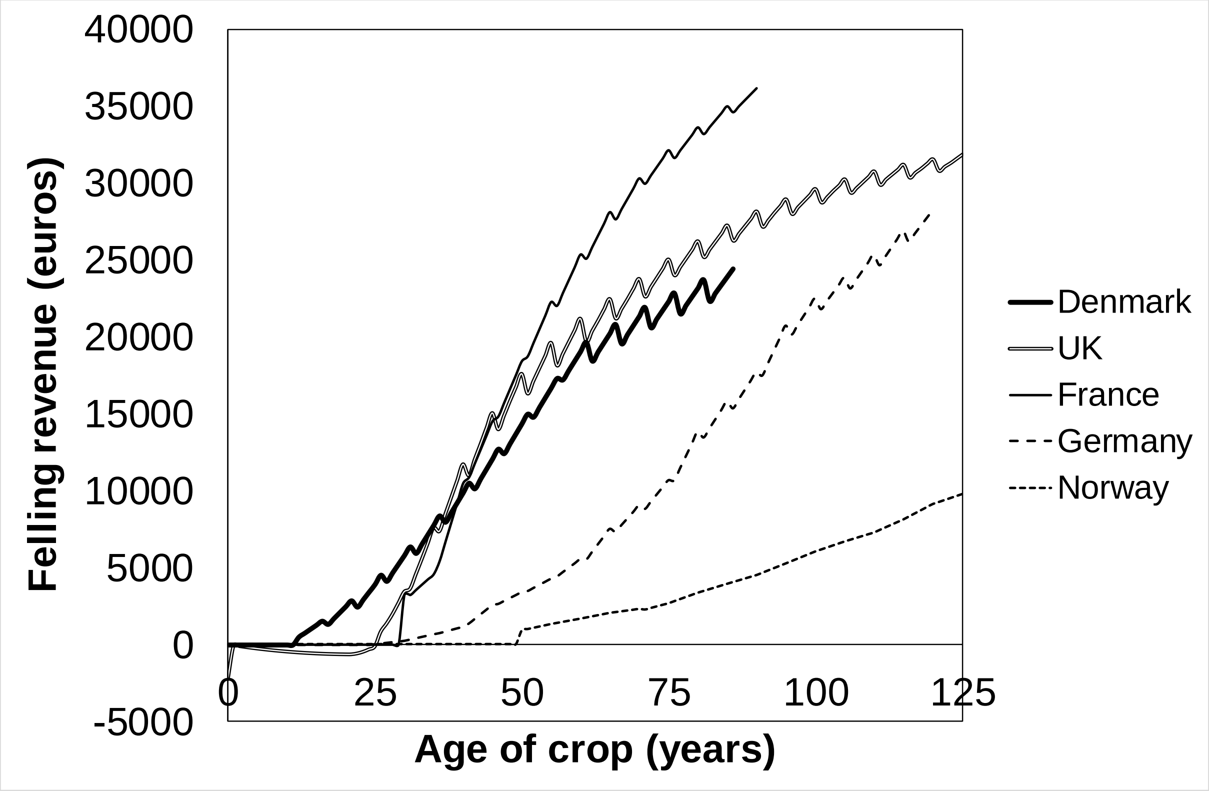

Figure 1: Felling revenue profiles through time

- Arbitrarily, a sequence of seven crops was taken. No methodological problem would be created by increasing this number, but it was not necessary: the seventh crop always began after – sometimes long after – the chronological time when discount rates had become constant in each of the countries' schedules. Thus for the seventh and all its successor crops, the rotation could be optimised according to the standard Faustmann formulation.
- For the sixth crop, starting initially after 1000 years, the NPVs of each possible rotation length, at their provisional locations in chronological time, were calculated. The discount factors for each cash flow were those appropriate to its provisional location in time. For each possible rotation length, the seventh crop was initiated immediately after this rotation's end. The NPV of the seventh crop and all its successors was thus included, the discount factors being again those appropriate to the time of initiation. The rotation giving maximum overall NPV, for the sixth and successor crops, was the provisionally optimal one for the sixth crop.

- The same procedure was adopted for the fifth, starting initially after 800 years, then for successively earlier crops, the fourth, third, second and first, starting initially after 600, 400, 200 and 0 years. Each evaluation included the value of initiating the following succession of crops immediately at the end of each of the current crop's possible rotations. Once again, in each case the discount factors used for each cash flow were those for its provisional location in chronological time.
- This process was repeated backwards for each crop until the first.
- The provisional optimal rotation for each crop affected the position in chronological time of all its successors. This altered the profile of discount factors applicable over those crops' own rotations. Hence the whole provisional optimisation process was iterated using the newly applicable discount factors.
- A stable solution normally arose within 20 iterations (Price et al., 2017) – much more quickly than with previous algorithms. Occasionally, however, hundreds of iterations were needed for stabilisation.
- This algorithm was applied to every combination of country cash flows and discount schedules. The results were noted wherever they illustrated a point of potential interest.
- A discount rate that varied continuously rather than in steps was also used extensively. Its profile approximated that for the UK's discount rates, declining from 4% asymptotically towards 1%, according to the formula:

  $[\text{discount rate}] = 1\% + (4\% - 1\%) \times e^{(-0.008 \times [\text{time}])}$

- Note that the Faustmann formulation for the optimal final rotations is not strictly correct for this continuously declining rate. However, in practice the single discount rate used to optimise the seventh and its successor rotations was close to the asymptote of the declining rate: rotations were approximately the same as those for the asymptotic rate. Moreover, the cash flows from the seventh and following rotations contributed negligibly to the overall LEV.
- One caveat should be noted about the protocol described: at the identified optimum for a given crop, the discount rate profile over this crop's own rotation is precisely correct, for all possible crop ages. If the present rotation is shortened or lengthened, however, for *subsequent* crops the crop age is commensurately displaced relative to the discount rate profile, slightly affecting their calculated NPVs. Despite this potential problem, both theoretical analysis and numerical experiment showed that the peak of NPV at the identified optimum is actually sharpened when adjustment is made for this imprecision: the optimal rotation is correctly identified by the protocol, and its NPV correctly calculated.

  Table 2 shows the iterative evolution of results

Table 2: Results of a few iterations, for UK data, with continuously declining discount rate, and an imposed minimum 50-year rotation

	Rotation schedule after the following iterations									
Crop	Initial position	After 1st	After 2nd	After 3rd	After 4th	After 5th	After 6th	After 7th	…	Final equilibrium
I	200	50	50	50	55	55	55	55		55
II	200	60	50	51	55	60	59	59		58
III	200	70	50	55	60	62	62	62		60
IV	200	72	55	58	62	65	65	64		64
V	200	72	62	60	65	69	69	69		69
VI	200	73	65	65	70	72	72	70		70
VIIff.	200	73	74	74	74	74	74	74		74

Some indicative results: discount rate and rotation

Results were generally sensible and stable, with rotations lengthening up to the time when the discount rate became constant. This accords with the standard result for discount rates constant through time, that the lower the rate, the longer the rotation (Price, 1989). Figure 2 gives an example: Norwegian cash flows are discounted using the UK's stepped schedule. Successive rotations lengthen to 125 years, as the discount rate declines, until 300 years, when the discount rate reaches its final value of 1%.

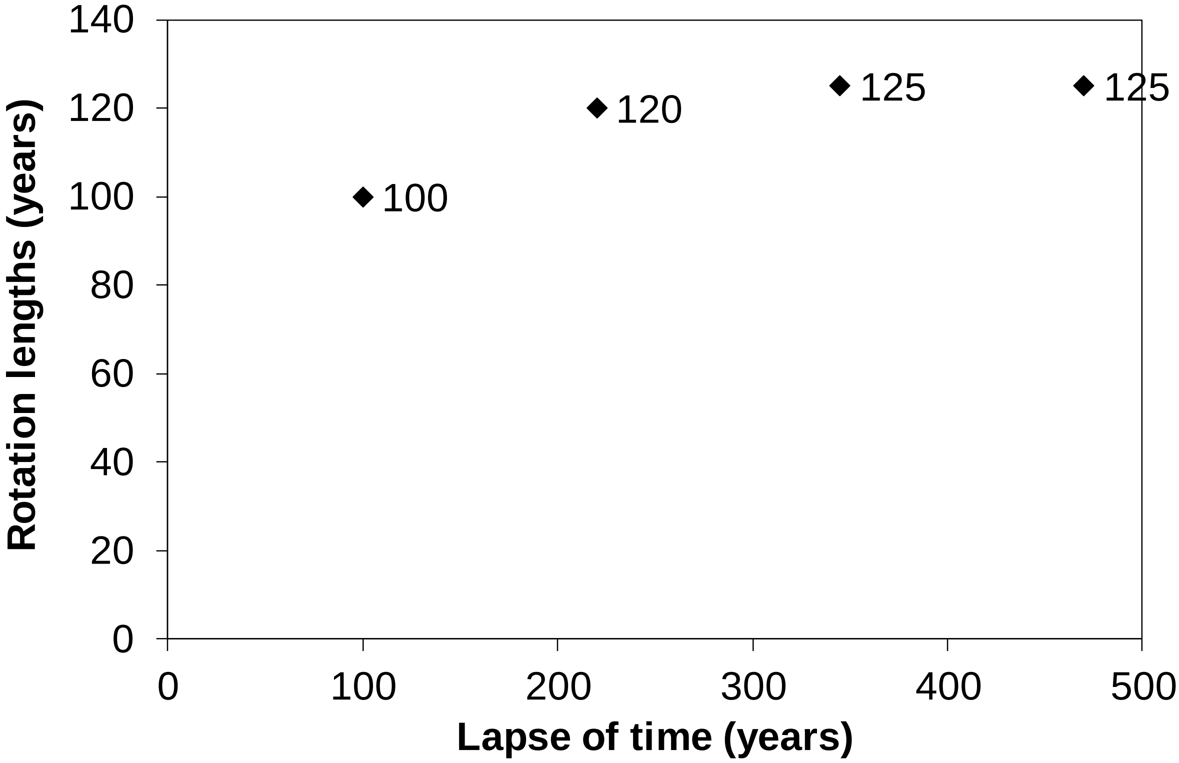

Figure 2: Norway spruce productivity 3.5 m^3/ha/year; Norwegian revenues; stepped UK discount rates

For this case, and whenever the result was tested, the optimal sequence was identical, irrespective of provisional starting conditions, for example with initial rotations of either 50 or 200 years. This is procedurally advantageous, compared with a backwards recursive algorithm, which needs to set the starting condition with a longer rotation than any individual crop's optimal rotation.

With a constant discount rate (e.g. Germany's 4%), the optimal rotation for each successive crop, as calculated by the global search algorithm, was identical. It was also the same as that derived by routine application of the Faustmann formula: this provides a procedural validity check on the solution algorithm. For interest, LEVs in each country's own currency are shown in table 3, mostly negative as a result of Germany's high discount rate and each country's crop formation costs, which were in the range from €2000 to €4000.

The fourth and fifth columns show results for France's discount schedule, which uses 2% for years beyond 30: all crops have the same optimal rotation, because the discount rate has fallen to its final value of 2% before the end of the first rotation. The LEV calculated from the initiation of any of the second crops is here much higher than that seen from the beginning of the first: for example, for French cash flows the LEV changes from €2646 for the first crop and its successors to €7805 for the second crop and *its* successors. The change occurs because the period from year 0 to year 30, during which the relatively high 4% discount rate is used in France, no longer has influence, when calculations are done from the initiation of the second crop. €7805 is also the LEV when a uniform 2% discount rate is used from the beginning. Such conformity with reasonable expectations further enhances confidence in the search algorithm's effectiveness.

Table 3: Rotations with Germany's 4% discount rate, and with France's declining schedule

Cash flow	German 4%		French 4% then 2%	
	Optimal rotation (years)	LEV	Optimal rotation (years)	LEV
German	95	€–1252	112	€791
Norwegian	82	NOK–5968	120	NOK–2376
Danish	54	DK731	70	DK24,091
French	56	€–900	65	€2646
UK	55	£1455	64	£4989

During periods when schedules prescribed high discount rates, rotations were generally shorter than when schedules prescribed lower rates. Figure 3 compares results of applying the UK's and Norway's discount schedules to French cash flows. The UK's stepped discount rate is higher than Norway's between 75 and 120 years, giving shorter rotations, but the UK's is lower after 200 years, giving longer rotations. Further influence, discussed later, arises from the different calculated values of bringing successor rotations earlier in time.

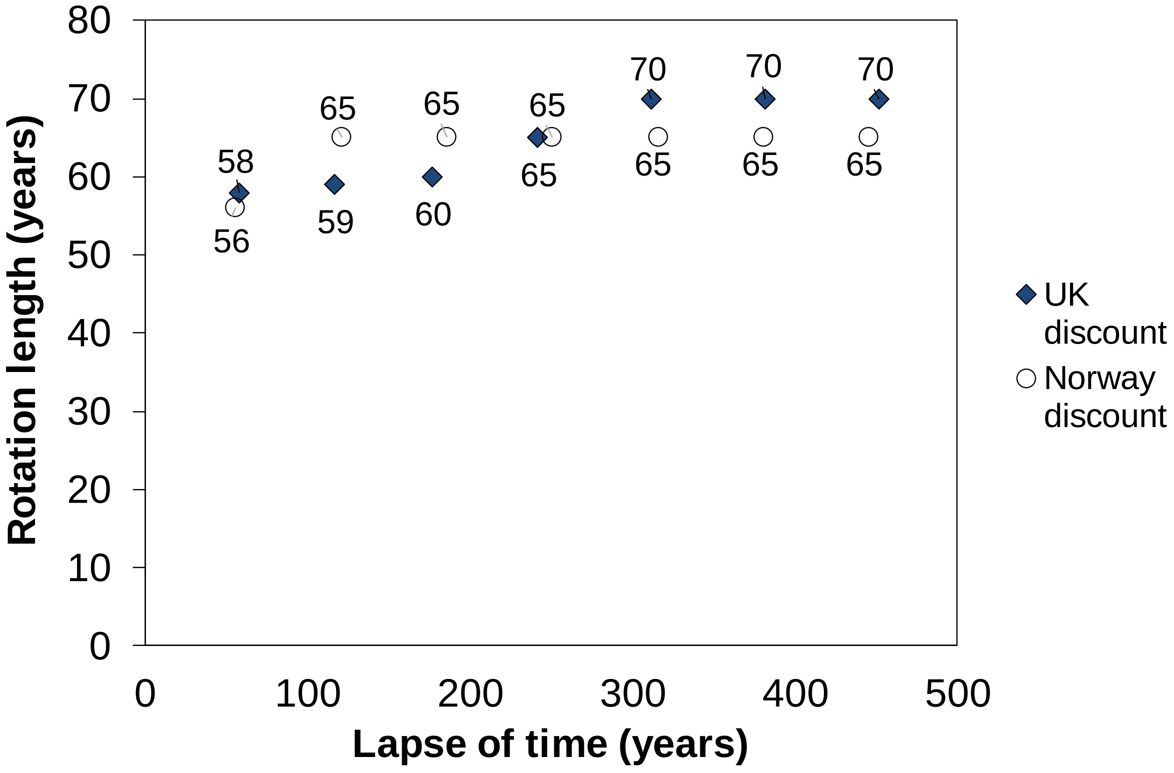

Figure 3: Rotation sequences using French cash flows together with UK and Norwegian discount schedules

Rotation effects of earlier and later discount rates

The strong influence of whatever specific discount rate applies when a rotation is reaching optimum is expected, and has been demonstrated above.

No cases were expected where an earlier discount rate would affect current rotation. For example, using French cash flows, after 75 years when French, Danish and Norwegian rates are all 2%, all subsequent rotations were 65 years, irrespective of the rates earlier in the profile. Only the first rotation differed, at 56 for Norwegian and Danish discount schedules, which had not stabilised by the end of the first crop's rotation. With the French discount schedule, which stabilises at 30 years, optimal rotations were 65 years throughout. Customised modifications of the Norwegian schedule, setting the discount rate until 50 years as 1% or 10% had no effect on any of the rotations, except that the first rotation was shortened to 40 years under a 10% discount rate for the first 50 years. It might be thought inevitable that earlier rates could not influence later rotations. However, a customised revision of the UK's discount rate in which 1% prevailed from 0 to 100 years not only extended the first rotation from 55 to 97 years (expected), but also lengthened second and third rotations and shortened the fourth rotation, all of which terminated beyond the altered discount period (see table 4). This occurred because the prolonged first rotation shifted later rotations into different discount zones.

Table 4: Effect of early discount rate on later rotations: UK cash flows and discount rates

Normal rates			Customised rates		
Lapse of time (years)	Discount rate prevailing	Rotation (years)	Rotation (years)	Discount rate prevailing	Lapse of time (years)
55	3%	55	97	1%	97
110	2.5%	55	60	2%	157
168	2%	58	64	1.5%	221
232	1.5%	64	55	1.5%	276
287	1.5%	55	73	1%	349
360	1%	73	73	1%	422
433	1%	73	73	1%	495

Note: the order of columns is reversed between the two halves of the table in order to facilitate comparison of succeeding rotations.

Later discount rates may have a strong effect on earlier rotations. Figure 4 demonstrates the effect by lowering the Danish discount schedule after 140 years from 2% to 1%. The dashed line shows the time after which discount rates differ. Lowering the future discount rate dramatically reduces the previous crop's rotation (from 70 to 50 years), because of the much-increased opportunity cost of delaying successor rotations. This has even exerted an effect on the first crop's rotation. The effect is yet more pronounced with a 0.5% discount rate in force beyond 140 years, the first, second and third crop rotations then shortening to 43, 38 and 38 years respectively, and subsequent rotations lengthening to 86 years (the longest included in the yield model). Where a low discount rate is in force, the opportunity cost of successor crops becomes the main factor curtailing rotations, just as happens with constant discount rates. However, when the period of low discount rate is distanced from the rotation under consideration by a period of high discount rates, this curtailing influence is mitigated.

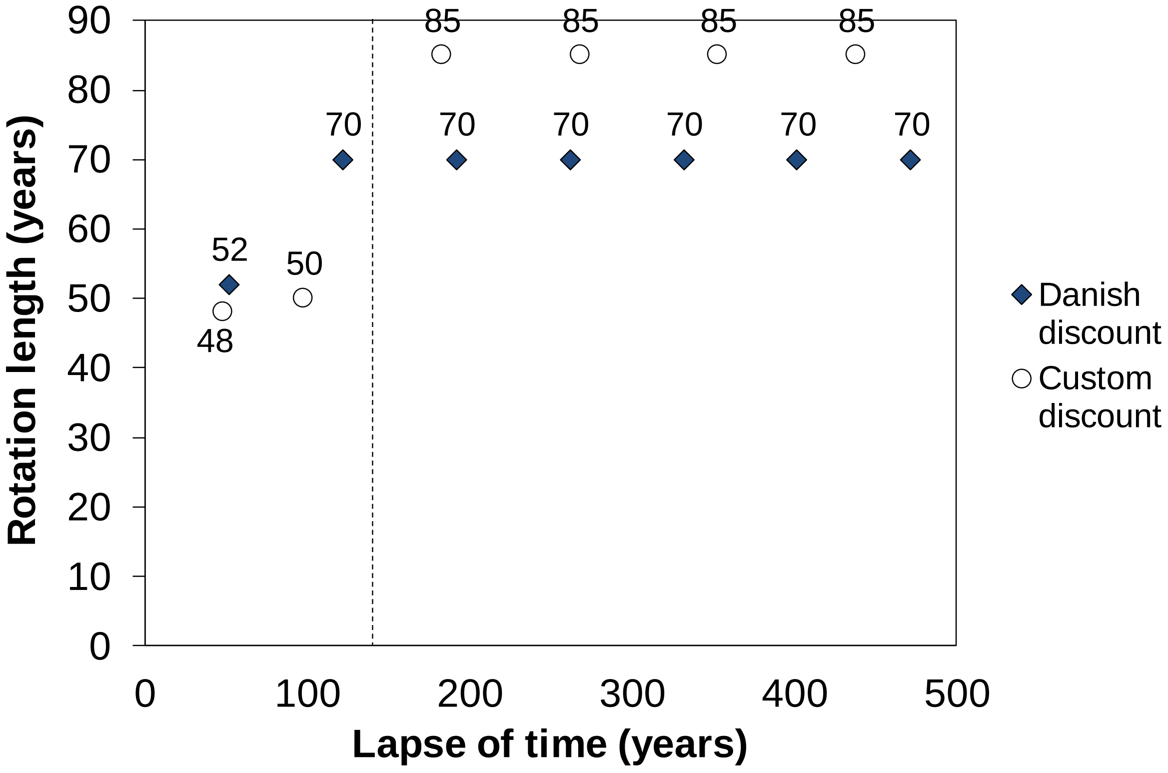

Figure 4: Danish cash flows discounted according to the normal and the customised Danish discount schedules

Productivity, crop formation and thinning effects

In accordance with results for a constant discount rate, higher (financial) productivity crops tended to have shorter rotations for a given discount schedule. Table 5 records results using the continuously declining schedule.

Table 5: Effects of crop productivity on rotations

Country cash flow	Productivity (m^3/ha/yr)	Crop 1st	2nd	3rd	4th	5th	6th	7th
Norway	3.5	100	120	120	122	125	125	125
Germany	8	97	114	119	120	120	120	120
France	12	58	60	65	65	70	70	70
UK	16	55	58	60	64	69	70	74
Denmark	20	54	56,57	60,62	69,70	74	80,81	86

For later crops, Denmark has longer rotations than the UK and France, despite greater physical productivity. This is because of differences in price profile. As figure 1 displays, the UK's revenues rise further, and France's revenues rise *much* further, than Denmark's. Such high revenue tends to shorten rotations, just as when the discount rate is constant (Johansson and Löfgren, 1985). The alternative rotations shown for Denmark's second, third, fourth and sixth crops reflect the slight oscillations arising during iterations of the optimisation algorithm, as discussed below.

Lower crop formation costs shortened rotations. For example, under the Norwegian discount schedule, costless crop formation by natural regeneration decreased French rotations, from 56 to 52 years for the first crop, and 65 to 61 years for the second and subsequent ones.

This is shown in figure 5. The Danish discount schedule produced a similar result. With the constant 4% German rate, all rotations were 51 years with costless crop formation, and 56 years without it. Again, such changes are the standard result under constant discount rates.

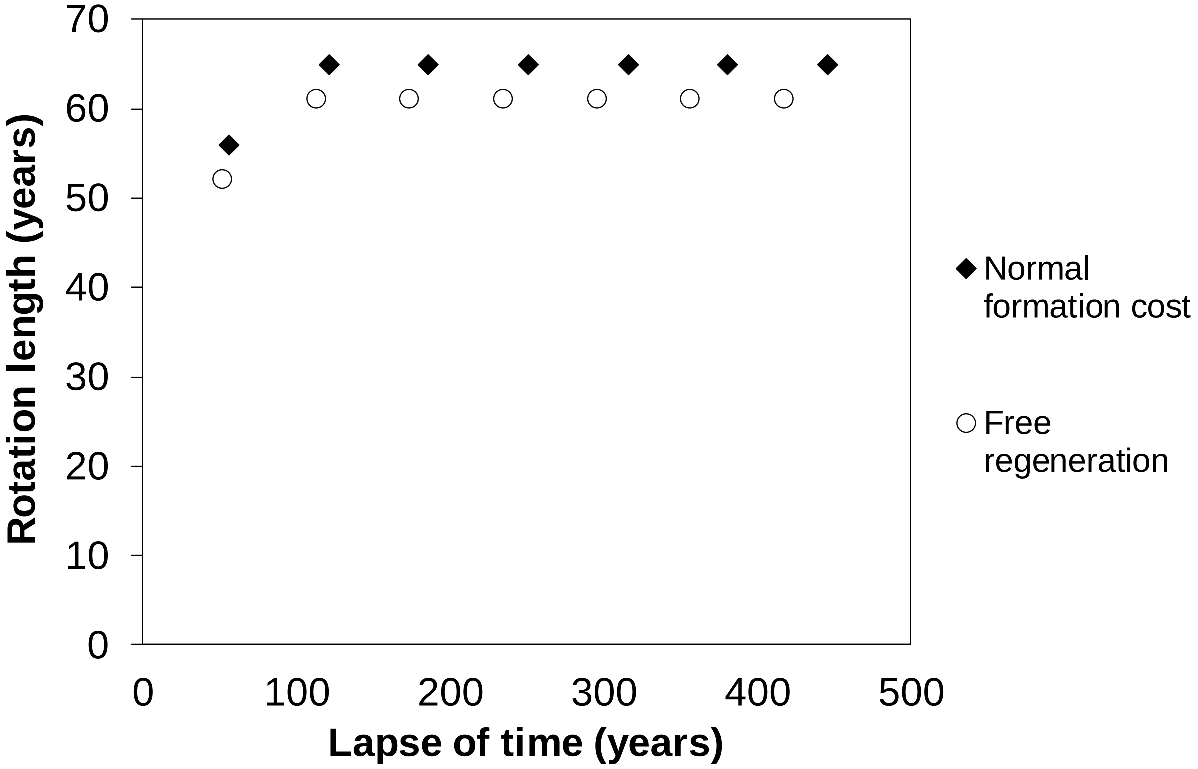

Figure 5: Norway spruce productivity 12 m^3/ha/year; French cash flows; Norwegian discount rates

A notional yield model was created in which all thinning revenues for the Danish cash flow, instead of arising at the scheduled time, were aggregated with final felling revenue. (This had been done with an earlier-used algorithm, to circumvent problems of local optima (Price et al., 2016).) The results are displayed in figure 6. When potential thinning revenues were aggregated with final revenues, rotations were 7 to 15 years shorter. This is because the absence of “timely” thinning revenues makes it more urgent to secure the aggregated revenue, which is only available at the end of the rotation. Similar results are usually found under a constant discount rate: with the German 4% rate, the Danish “unthinned” rotation is 45 years, while the thinned one is 54 years.

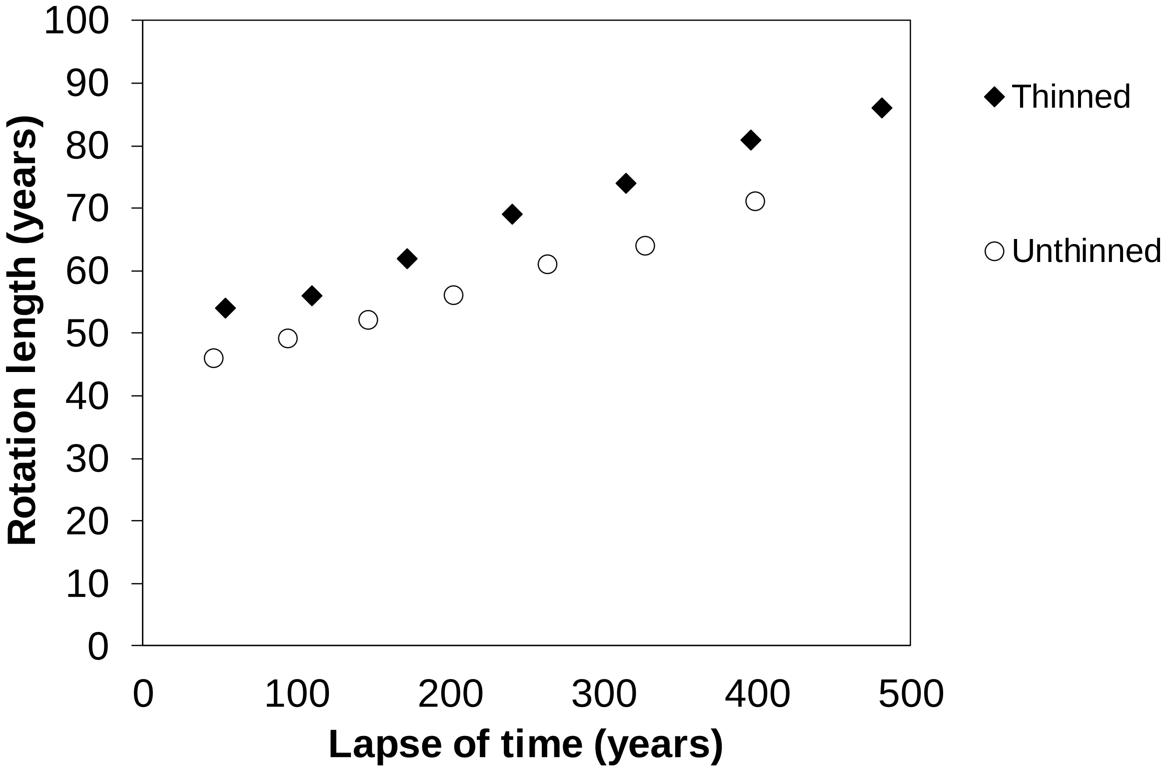

Figure 6: Danish cash flows, smoothed discount schedule, with thinning revenues included at the scheduled time ("thinned") and reallocated to the end of the rotation ("unthinned")

Anomalies

With the UK's discount schedule, which declines stepwise to a minimum of 1% over a 300-year period, apparent anomalies often arose, with some crops having a shorter rotation than either the previous crop or the successor crop. Figure 7 shows the result for UK cash flows, with costless regeneration.

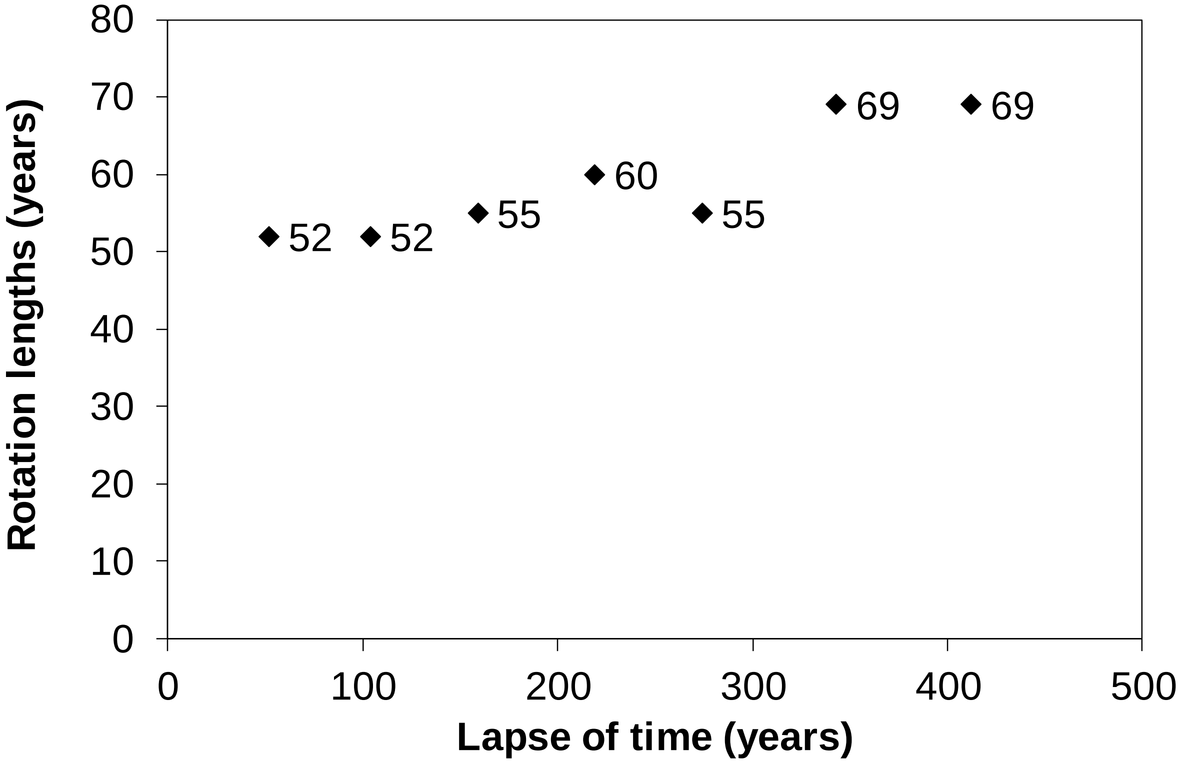

Figure 7: Anomalous fifth crop rotation for UK cash flows, costless regeneration, UK stepped discount schedule

The effect was commonly displayed for the crop rotation ending shortly before a downward step in discount rate, particularly before the final step down, from 1.5% to 1% after year 300. The anomaly also appears for Danish and French cash flows. The result arises from the shortening influence exerted by the high opportunity cost of immediately succeeding crops, when that is subject to a low discount rate. The effect was mitigated by using the continuously declining discount schedule, as shown in figure 8.

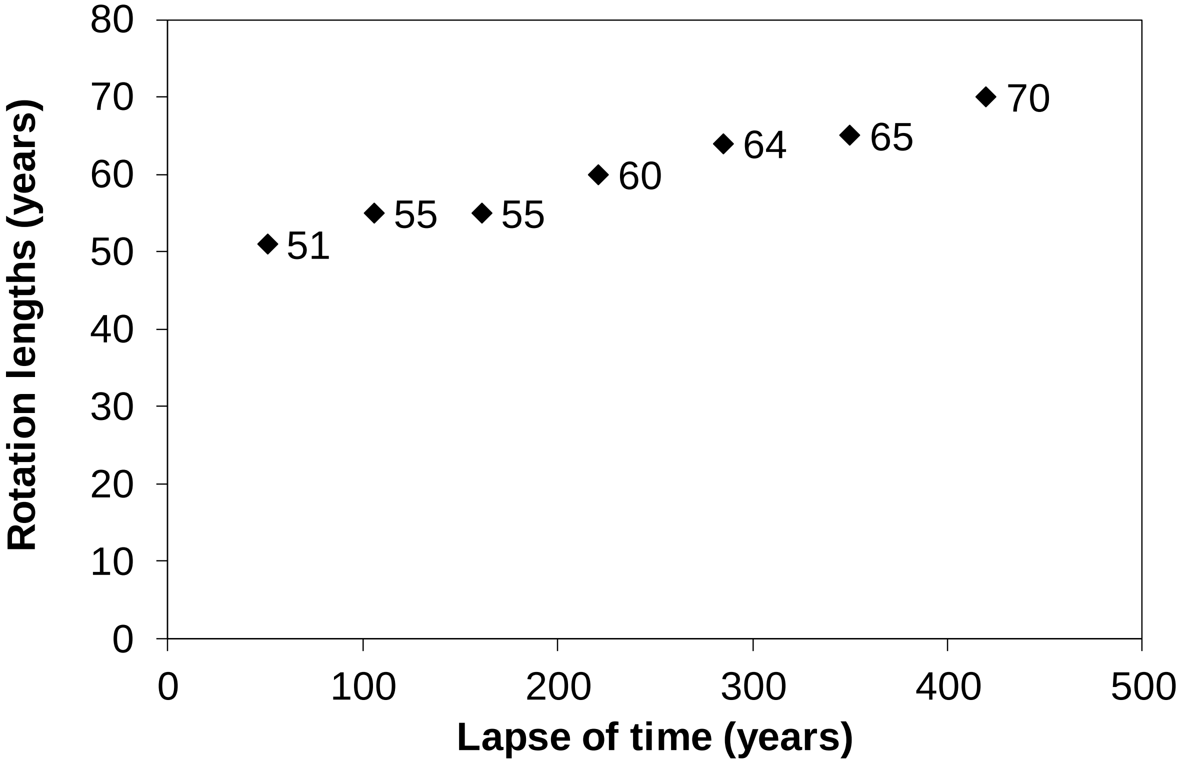

Figure 8: UK cash flows, costless regeneration, continuously declining discount schedule, no anomalous rotations

Oscillations

The UK Treasury's discount schedule is the one that declines over the longest period (300 years), and takes the greatest number of steps down (five). These characteristics seem to make it liable to cause oscillating solutions. Big oscillations occurred for German cash flows, the second crop lunging from 95- to 115-year rotations and back. In table 6 are shown the four configurations arising cyclically. Manual insertion of each of these configurations allowed stable results to be generated and compared. Oscillation between 95 and 115 years might hint that an intermediate rotation, say 105 years, would produce a result superior to either. However, manual insertion of this rotation showed an LEV which was inferior to that at 115 years. Experiment with non-candidate second rotations showed a marginally superior LEV at 117 years.

Table 6: Candidate rotations for first, second and third crops (later crops all have 120-year rotations)

	Rotation (years)					
First crop	97	96	96	97	97	97
Second crop	95	95	115	115	105	117
Third crop	120	120	120	120	120	120
LEV of all (€/ha)	–77.2	–79.4	–71.2	–68.5	–73.5	–68.4

The continuously declining discount schedule gave a stable result, as shown in figure 9.

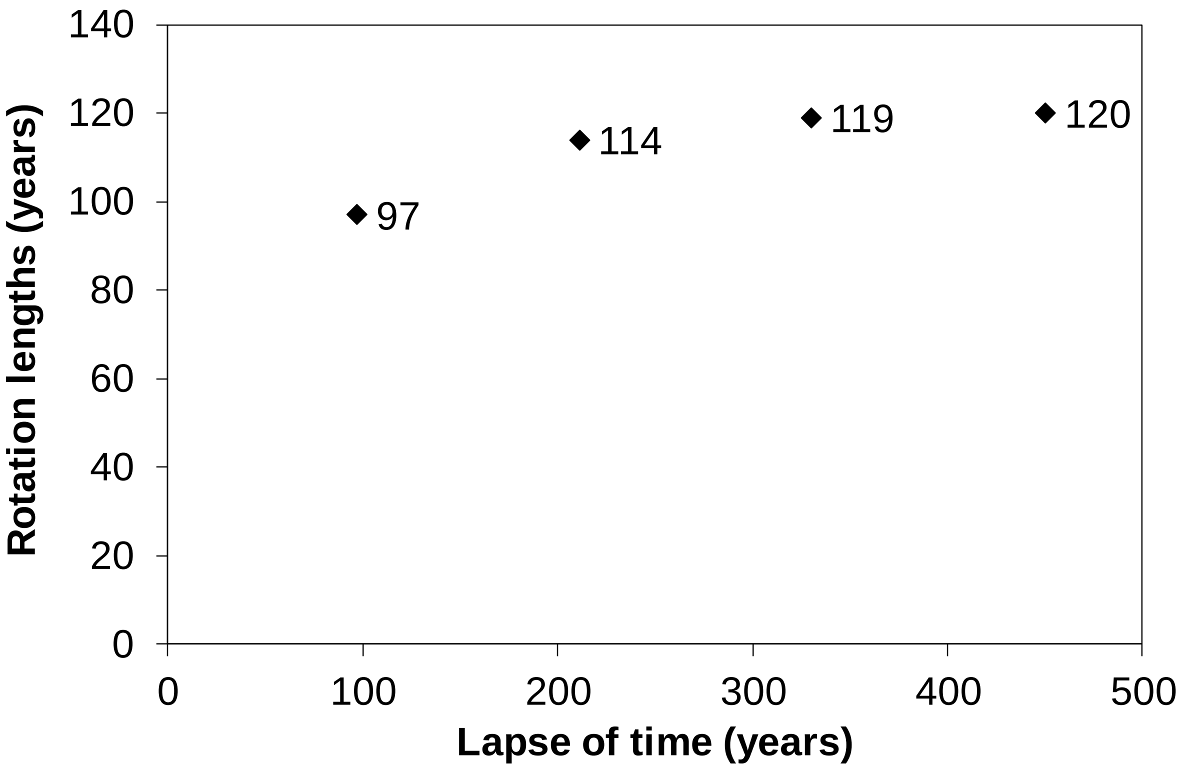

Figure 9: German cash flows, normal regeneration cost, continuously declining discount schedule, no oscillations

Smoothing the irregularities of final felling revenue, by taking 11-year running means for the German data, also produced a stable result.

Major oscillations with the stepped discount schedule also arose for Denmark's cash flows: for the fourth crop's rotations (from 63 to 69 and back) and for the fifth crop's (from 50 to 86 and back). Discounting with the continuously declining schedule greatly reduced the oscillations: the second, fourth and sixth crops oscillated by a single year, and the third crop by up to 2 years. None of these configurations produced markedly different LEVs. With free regeneration, there were no oscillations. For France's cash flows, the second crop's rotation oscillated between 58 and 59 years. No oscillations at all occurred for the UK's and Norway's cash flows.

The oscillations happen because rotations are calculated sequentially. Each crop's rotation is optimised, given the provisional rotations of later crops, which themselves are revised when the earlier rotations have been altered.

Other oscillations were found only with the Norwegian discount schedule: the first crop using Danish cash flows oscillated between 52- and 53-year rotations; the first crop using UK cash flows oscillated between 53- and 55-year rotations.

Conclusions

Earlier attempts to find an algorithm for determining the optimal sequence of rotations had encountered several problems. The algorithm described above:

- used global search within each crop's rotation rather than a first-order condition, so avoided stopping at local optima;
- gave results which were independent of starting conditions;
- included thinning revenues and crop formation costs, at the time in the crop cycle when they actually occur;

- allowed a perpetual sequence of crops to be included – not of much practical significance, but making the solution tidy; and
- delivered results from use of different inputs that paralleled those found in conventional optimisations using the Faustmann approach.
- It was demonstrated that results (rotations and LEVs) differed substantially from those obtained under a constant discount rate.

The results may be considered reliable for the conditions described. Seemingly anomalous results were actually explicable, arising from increasing domination by the opportunity cost of successor rotations as time progressed. Oscillating results were attributable to stepwise discount or irregular revenue functions: they usually disappeared when functions were smoothed. These results are in any case of small practical significance, as LEVs were very little changed over whatever oscillations were found.

The differences in rotation and profitability between results obtained under constant rates and those under declining ones are not trivial. For example, using Norwegian cash flow data, the smoothed UK discount schedule gave rotations ranging from 100 up to 125 years and LEV of NOK–3972, while the constant German discount rate gave all rotations as 82 years, with LEV of NOK–5968. Both discount schedules use 4% as an initial rate.

Further discussion of the differences in outcome between constant and declining rates is given in Price (2011), which also shows the cost of using the “wrong” criterion to determine the rotation sequence.

The spreadsheet used for the calculations above is still under development, with the objective of making it easier to use. A copy of its current state is available from the corresponding author.

The solution algorithm could be modified to include other kinds of cost and benefit flow. A single column of annual aesthetic values (increasing with crop age) could readily be included in the protocol as it exists, with an additional element added to a crop’s NPV, in the same way as crop formation costs and thinning revenues are added. Carbon values would raise greater problems, owing to the diversity and time lags of carbon fluxes (Price and Willis, 2011), and to the fact that social prices of carbon vary through time with a declining discount rate schedule (Price, 2017b).

The expectation is, that including such benefits would prolong all rotations, compared with rotations determined without including them. However, in other respects the results would be similar to those given above.

Acknowledgements

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Appendix: Technical note on the spreadsheet's structure and operation

- Cash flow data for all included countries are in the sheet `CashflowData`. More yield models could be added in the same format. Note that final revenue is that available *before* thinning. Depending on how country data are recorded, some adjustment may be needed from the national financial model.
- The yield model is selected in cell B2 of the sheet `Working`. Data for the country yield model selected take values from columns A, C & D in `CashflowData`.
- These columns become columns D, E and F in `Working`.
- Similarly, discount schedules are in `DRsource`; the operational rate selected in cell M5 of `Working` copies the appropriate rates to column N of `DRsource`, which then becomes column B in `Working`. If the smoothed discount schedule is selected, the parameters of the schedule are set in cells F6, H6 and H7 of `Working`.
- There are two sets of discount factors in `Working`. Column C is a data bank in which to look up factors resulting from overall time lapse which is in column A. Column H shows discount factors within the current crop's cycle, discounted to the beginning of the cycle, but using the set of discount rates appropriate to the crop's provisional location in chronological time. The rather strange hybrid has been enacted so that discount factors do not have to be separately compiled from the beginning for each cash flow. The discounting process is brought to time zero by a further amendment, using the discount factor from the beginning of the crop to time zero, when LEV is calculated.
- If any cash flows are recorded in column E, their values discounted to the crop's initiation time are given in column I.
- In column J the LEV is compiled from (a) the crop formation and thinning DCFs up to but not including the current year; (b) the discounted felling revenue if felled in the current year; (c) the discounted value of successor crops on their provisional rotations, launched from the current year. All these are discounted to the beginning of the current crop's rotation. The rotation for the current crop with highest LEV is identified as the optimal rotation, given the rotations provisionally adopted for all successor crops.

- When the current crop is the first one, the calculated values are discounted to the beginning of the current crop's rotation, i.e. time 0. The provisional optimal sequence of rotations is that giving highest LEV.
- When the highest LEV is found for each crop, it is marked in column M, with the corresponding rotation in column N.
- Column L is used only for LEV of seventh and following rotations, in rows 1224 to 1424.
- Column K (usually hidden) is just a matter of report, for the discount rate corresponding to the lapse of time in column G.
- Cells A2 to M10 are for defining inputs.
- Cells O2 to O7 allow, if desired, rotation lengths for any of crops 1 to 6 to be "forced" to take a given value. This facilitates LEV comparison of different configurations of crop rotations, particularly when the optimisation protocol results in oscillations of both rotations and LEV.
- Cells Q2 to S8 record the results for successive crops.
- Rows 12 to 212 are for the cash flows of the first rotation, in so far as they exist.
- Rows 214 to 414 replicate the rotational cash flow data, for the second rotation.
- And so on for the third to sixth rotations.
- The seventh rotation is representative of itself and all successor rotations, so is treated differently in the discounting phase, using the usual multiplier from the NPV of one rotation to the LEV of a perpetual sequence of rotations. The discount rate is that applicable to the year of initiation of the sequence. By this time the discount rate will have stabilised (or for the continuously declining discount rate, be close to the asymptote).
- Once the provisional optimum has been identified in each rotation, the crop is terminated and any subsequent cash flows are ignored.
- In column G the time lapse from time 0 is set, such that the current crop is initiated at the provisional termination time of the previous crop.
- In this manner, successive crops are "stacked up" against the lapse of chronological time. With each iteration, the stack is amended, with each crop relocated in chronological time, until a stable result is obtained.

This rather convoluted process is necessitated by the changing location of each successive crop in chronological time, and thus the discount profile applying to it.