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Master thesis

Attention and price variation in the U.S. equity market: Is there an Elon Musk effect?

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Preface

First of all, we would like to extend our greatest gratitude to our two supervisors, Erik Haugom and Štefan Lyócsa, for all the help and tips they have given us regarding data collection, analysis, and writing. Both Erik and Štefan have helped tremendously. We are grateful that we were lucky enough to have them as our supervisors, as they both have such great knowledge and experience in the academic field. Further, we would like to thank our families and friends for their patience, help, and corrective reading. These last two years have been busy, and we have worked many hours to get to where we are today. We both stand proud as another chapter is closing in our lives.

Thanks to Inland University for having us, a time that we look back on with smiles on our faces. We have met new people and friends. We were also the first class to attend this master's degree program and the first to finish it. We look back at great classes, with even better lecturers.

We want to thank the scientific committee and the conference chairs at Lillehammer Business Analytics Workshop 2024 for the opportunity to present our work at such an event. It is a truly great honor for us.

We also want to extend our condolences to anyone affected by Gudbrand Liens' passing. Unfortunately, he left us way too early. He was a great lecturer, and we were happy with the course he taught and learned a lot. As the dean for our degree, he was always looking for ways to better the way for us and later classes and was always interested in our opinions and feedback.

Again, thank you so much to everyone who has been helpful throughout the degree, and this thesis. We proudly look back on two years that went by quickly and are grateful for our new friends and connections.

Tom Inge Grønlien — Simon Alexander Oppegård

Abstract

This thesis examines whether Elon Musk's Twitter use is associated with Tesla's stock price volatility by examining the number of tweets, retweets, likes, and replies his activity generates.

These aggregated numbers are then tested against how his Twitter activity and engagements with the public via the platform impact Google search volumes. Both of these factors are then tested against different range-based volatility estimators.

We used the range-based estimators from Rogers and Satchell (RS), Garman and Klass (GK), and Parkinson (PK). These estimators calculate the volatility regarding day-to-day changes. During the data exploration portion of the research, significant autocorrelation lags were discovered in the Google search volume for "Tesla."

The key findings were that Elon Musk's use of Twitter led the Google search volumes of both keywords "Elon Musk" and "Tesla" and that his use of Twitter was statistically significant towards the volatility estimators. However, the observed changes were smaller than first anticipated but they confirmed that there is an effect. In some specifications, we identify that there is a significant effect of social media interest, but this result is not consistent across all specifications; this might be because of how the HAR-RB captures the realized volatility.

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1 Introduction

1.1 Background for the thesis

The inspiration for the thesis came from the article "How tweets by Tesla's Elon Musk have moved markets" by Nadeem (2021). The article lists times when public statements or tweets by Elon Musk affected the stock price of Tesla or other companies mentioned. Every incident resulted in a rise or drop of several percentage points in the company's stock price, which was mentioned in Musk's tweet. Another example from the article is that Musk tweeted that he thought the Tesla stock price was too high, and within the same day, Tesla lost about 13 billion dollars in market value. This list sparked research interest on whether and how Elon Musk can control or manipulate the stock market or specific stocks.

This thesis explores whether Elon Musk's Twitter activity influences stock market volatility[1]. Specifically, we examine both Elon Musk's Twitter activity and Google search volumes related to Tesla and Elon Musk against Tesla's stock price volatility. The thesis will focus on whether there is an "Elon Musk effect" on the U.S. equity market. The "Elon Musk effect" is simply said to be that his use of Twitter or interest towards himself is associated with future Tesla price movements.

In contrast to the other research on this subject, we will not focus on semantics, what he was saying, and how it was said. Nor will we focus on whether the stocks rose or fell. We will focus on whether he is saying anything at all and how it affects attention and volatility in asset pricing. This is particularly interesting for smaller investors utilizing volatility in their investing and for larger institutional investors who base their investments on volatility. Anecdotal evidence suggests that his public and social media interactions move markets, especially cryptocurrency markets. What about equities?

In particular, this thesis will investigate if Elon Musk's apparent advantage in the stock market is also present in the volatility of his stock in Tesla. There have been numerous news articles and even legal actions regarding his Twitter use for stock price manipulation. Elon Musk is the CEO of Tesla and SpaceX and has more than 150 million followers on

[1]Recently, Elon Musk changed the name of Twitter to X, which will be referred to as Twitter throughout this thesis, because X could be misinterpreted for an unknown or something else.

social media. This enables him to reach the whole world within seconds. He also acquired Twitter in October 2022 after months of lawsuits and verbal mudslinging (Conger & Hirsch, 2022). As of the 15th of April 2024, Musk is also the second richest man in the world, with a stunning net worth of 180 billion USD (LaFranco et al., 2023). He includes himself in different public discussions, and he does not hesitate to release his statements publicly, often through social media or Twitter, more specifically, where he joined back in 2009.

Tesla is a car-brand company that manufactures their own electric vehicles. Tesla is registered at the NASDAQ stock exchange in New York, and the company went public in 2010 (Jarslett, 2024). Tesla was founded in 2003, and Elon Musk was one of the early investors. He is the most known of the co-founders of Tesla and is currently the CEO of the company. They have several car models, whereas their newest model, the *Cybertruck*, has gained a lot of media attention in the last year due to different attributes, like that it is bulletproof.

We therefore find it interesting to examine whether Elon Musk‘s Twitter use affects the volatility of his Tesla stocks, as stock volatility is used to assess the risk of investing in them. This is of both practical and academic interest.

1.2 Equity markets

Four centuries ago, in 1602, the Amsterdam market for the Dutch East India Company opened, and it is considered the first market where modern securities were sold (Petram et al., 2011). This can be considered the first modern exchange in history, similar to what we consider and know to be an exchange today. From that point, there have since been established several more exchanges, and today, almost every country has one. The larger and more well-known ones internationally include the New York Stock Exchange (NYSE), the Oslo Stock Exchange (OSEBX), and the Euronext exchange in Europe. These exchanges are marketplaces that sell financial instruments, such as equities, commodities, and bonds. Early on, the purchases of such financial instruments were executed by brokers, and everything had to be done through people working at the exchanges. In other words, if you wanted to buy a company's equity stake (stock), you would contact a broker to execute the purchase or trade on your behalf. In later years, this concept is slowly fading, as internet-based companies give investors the power to invest on their own through their services. This excludes the broker, as the purchase is done through the Internet and not physically as it once was. This can be more beneficial for the investor, as the costs of hiring a broker to execute the trade are no longer present. This cost is typically a percentage fee that the broker charges you for the trade (Barber & Odean, 2001).

Over the course of history, stock markets have undergone many crises, which have varied considerably in severity, duration, and, above all, in the causes that led to them. For example, in 1929, a global economic crisis was later named the Great Depression. This started with a stock market crash, resulting in higher unemployment, shorter life expectancy due to poverty, and greater cuts in GDP per capita across the world (Granados & Roux, 2009). The Great Depression lasted from 1929 to 1939 before the global economy stabilized. Furthermore, in 1900, we had the 1970s oil crises in 1973 and 1979, and the Asian financial crisis in 1997. The latter crises are noteworthy due to their consequences, but only hit certain parts of the world.

In the beginning of the new century started with a stock market crisis that later has been named the Dot-com bubble burst. The bubble burst in 2002 after the later years of the 90s experienced a speculative boom in most technology stocks, leading to inflated stock prices. This resulted in a significant decline in stock values and the collapse of many "dot-com companies". After this crisis, many researchers have studied how and why this happened, and concluded that much of the crisis can be traced to different psychological factors when investing, such as follow-the-heard investing and that standard rational nor behavioral models can explain how investors acted in the build-up of the crisis (Ljungqvist & Wilhelm Jr., 2003; Valliere & Peterson, 2004).

In 2005, the build-up for the next economic crisis emerged. This crisis is called the global financial crisis or the Great Recession. Further, the build-up came from the US housing market, where the increasingly good financial outlooks made workers, bankers, investors, and economists spend more, loan more, and invest more money (Stiglitz, 2009). This is a typical reaction to periods of easier access to financing and can be considered normal. However, such periods are also subject to considerable financial risks as people might get overconfident and less risk-averse. In the "get-rich-quick" environment, people spend and invest too much with greater risks. As the central bank kept the interest rate low and, at the same time, subsidized the housing market, people continued down the track, and the bubble eventually burst in 2007 and 2008, increasing poverty and bankruptcies across the U.S. and Europe. (Samuelson, 2011).

The stock market can be affected by e.g. macroeconomic news and the attention the news results in (Cutler et al., 1988; Sun et al., 2020), and other information that surfaces from the company. The efficient market hypothesis by Fama (1970) explains how an efficient market should reflect all available information. Social media postings are considered publicly available and consist of information, opinions, and other statements from the author. Tweets by Elon Musk can be viewed as news and information regarding the company. As he represents the company when he makes public statements. In connection to this, he should be held accountable for any statements or similar that he makes, whether it is about Tesla or not. Most larger companies also have their own social media accounts or pages. Nadeem (2021) mentions examples of this, where Elon Musk's tweets that he is considering taking Tesla private, among others can be seen as news. According to Cutler et al. (1988), the macroeconomic news about a company can only explain a fraction of the variance in the stock price. The question is, how fast the news is being priced in, and how.

Moreover, investors are continuously looking for an advantage in the market regarding better information or predictive capabilities. Simply put, better information could give the investor an edge and leave the investor with a higher profit. Further, if this thesis can present evidence that people themselves can be able to have the market react or stock volatility change, it could affect how future generations invest in equities.

1.3 Research question

First of all, The "Elon Musk effect" needs to be defined. By the "Elon Musk Effect" we mean how attention to Elon Musk influences Tesla's volatility in the stock price.

This is measured through his Twitter activity and Google search volumes towards his name and Tesla, henceforth referred to as attention markers. And how they tie in with the volatility in the stock, so the research question is:

"Is there an Elon Musk effect on the U.S. equity market?"

Where the Elon Musk effect refers to when the interest in Elon Musk can be associated with future price movements.

1.3.1 Hypotheses

Based on the research question, and the goals for this thesis, the specific hypotheses that are being tested are:

H_1: Elon Musk's Twitter use leads the Google search volume for "Elon Musk".
H_2: Elon Musk's Twitter use leads the Google search volume for "Tesla".
H_3: The volatility of Tesla's stock price is influenced by Elon Musk's Twitter use.
H_4: The volatility of Tesla's stock price is led by Google search volume for "Elon Musk" or "Tesla".

1.4 Overview of how the thesis is structured

The following section will be the literature review, where one can find the theoretical grounds for the thesis and similar research conducted by others, as well as explanations of variables and measures we use in our models. We move on to the methodology, how the work has been done, and how it could be replicated. We then present the results from the models, data, and research and briefly comment on any tables, models, or variables in 4 Results. We then discuss the results and findings, compare them to other research, and conclude the thesis and its research. Practical and theoretical implications for the research conducted in this thesis and directions for further research.

2 Theoretical foundation

In this chapter, prior research will be presented, and existing literature surrounding the research question and this study will be presented. This chapter will explain how the former research was conducted, and how this research will be applied in this thesis. Then the opposite way, how this thesis will help contribute to the former literature in terms of gaps or as an extension to the research. Additionally, research on more direct measures and models will be presented and further explained in the methodology chapter of this thesis. The object of this chapter is to present relevant theory, which is needed for the empirical analyses later in this thesis. Currently, social media has become a part of almost everyone's daily life, and as it has become as big as it has, it is natural to try and understand the impact it can have, both on people and the financial markets around the world.

2.1 Search process for relevant literature

The research for this thesis is somewhat new and innovative. To our knowledge, it has not been completed before, at least in terms of how one single person or public figure can affect the stock market through their sayings and the attention they gather. This made searching for relevant literature and finding relevant peer-reviewed articles in this field difficult. The search process was changed as we did not find any articles or research directly correlating with the research question. The focus shifted to similar research, such as studies with cryptocurrency, as the two have many similarities. Further, studies and articles on direct measurements and models are being used within the thesis. All measurements and models can be defended by former research or with studies that were directly interested in using models and measurements that we applied as well, e.g. volatility or heterogeneous auto-regressive (HAR) models.

2.2 Asset pricing

Publicly traded assets have prices that reflect the perception of that asset's current value for a given point in time. For public companies traded on exchanges, these prices continuously change, at the daily level, we often encounter reports on the opening, highest, lowest, and closing prices. That is the price when the exchange opens, the highest and lowest price throughout the day, and the price when the exchange closes. For public companies, the stake in the equity of the company that is being traded is referred to as a stock or share. Similarly, commodities are also traded at exchanges and work the same way as stocks. Further, you can also find information regarding market indexes and exchange rates at these stock exchanges. These prices are continually brought to our attention through daily news, TV, radio, and social media (Taylor, 2008). The traded prices can be denoted more formally as P_t, where P is the price of an asset (stock in our case) at time $t = 1, 2, ..., n$, where the time-index denotes as t. This is the simplest equation for asset prices and gives us the foundation for further analysis, like studying changes in prices or comparing two prices at different times in terms of returns. Given the price P_t (continuous) returns (R) are defined as $R_t = ln(P_t) - ln(P_{t-1})$. This captures changes for an asset's price from period t to period $t - 1$.

When discussing asset pricing, one must talk about the most known and used model for pricing assets, the capital asset pricing model. The model itself has been attributed to Sharpe (1964) and Lintner (1965) and further researched and explained by several authors throughout the years (Bodie et al., 2013). Simply explained, it compares the asset or portfolio to the whole market, to find the expected return. The advantage of CAPM is that it is easy to implement, and the equation itself is easy to calculate and understand.

$$R_{t,i} = R_f + \beta\left(R_{t,m} - R_f\right) + \epsilon_{t,i} \tag{1}$$

where i is a given asset, R_f is the risk-free interest rate, $R_{t,m}$ is the return that can be achieved by investing into the overall market (thus index 'm'), the $(R_{t,m} - R_f)$ is the market risk premium and the β represents the sensitivity of i^{th} asset return to the changes in the market, while $\epsilon_{t,it}$ capture the remaining unexplained variation in returns.

This model is of interest as it suggests that changes in an asset's price are a function of overall market prices as well, which suggests that the volatility of an asset should also reflect market-level volatility, an implication that we will use in our empirical part later as well.

Volatility or variance estimators, such as (Parkinson, 1980), Garman and Klass (1980) and Rogers and Satchell (1991) are helpful when trying to model variance and volatility. They will later be referred to as range-based volatility estimators, based on the definition given by Molnár (2012), that the range (from range-based) comes from using not only opening and closing prices but also high and low prices for the day. Practitioners typically use standard deviation as an estimate of volatility, as it is easy and fast to implement and to understand. They realized that an estimator that used not only high and low but also opening and closing prices had to be more precise than the previous estimators (Molnár, 2012). Simply put, by using more information about the price path, the estimator will be more efficient. The data used for this research is the stock price open, close, high, and low. Overnight returns are also calculated, and both of the measurements will be used. More estimators have been published, with smaller tweaks and changes. Further, in the article by Molnár (2012), they ran simulations and found that the "Garman-Klass estimator" returned the most accurate measurements. Former research, either uses the average out of the three measurements or all of them. Throughout this study, the average of Garman-Klass and Rogers-Satchell estimators will be used.

Volatility can be described as anything that is changeable or variable. The more the variable fluctuates over time, the more volatile the variable is (Daly, 2008). In finance, the volatility measure is commonly used to measure risk. In simple terms, risk can be defined as the unknown, where the outcome is considered negative. For this study, several variables can define and explain the risk associated with stocks and asset prices. An example of one of many of these factors can be the debt-to-equity ratio, where companies with more debt are considered more risky compared to others with less or without debt. Debt has to be paid back; if it is not, the consequences can be detrimental to companies. By practitioners, volatility is measured by the standard deviation of the returns. It is being used, due to its simplicity, even though the measurement itself is not as accurate and appears noisy compared to other volatility measurements. Such estimators will be introduced and further explained under the literature review section.

2.3 Literature review

In the preparation phase for this thesis, we dived into the existing literature to understand and find any gaps in the literature that our study could fill out and thus contribute to our understanding of price variation. Starting, we looked at the commonly referred to measurement of risk, and volatility, which is and has been a fundamental part of finance and econometrics. Volatility itself is a measurement of the fluctuations of the underlying asset price. Many investors and practitioners use it as a measurement of uncertainty. Further, this implies that it can be used as a measurement of risk in terms of valuation of investments, portfolio management, and similar (Poon & Granger, 2003). But even though it is natural to point towards finance and econometrics when discussing volatility, it is, on a larger scale, a description of how the economy and the markets move up and down. This again, does not only affect those who manage financial portfolios, but also everyone else, like regulators, legislators, lawyers, builders, homeowners, and others (Shiller, 1992). In simpler terms, volatility is the movement or fluctuations in, e.g., a stock price, market price, or GDP growth, depending on what you want to measure. Estimating volatility is essential when assessing investment strategies, constructing portfolios, and pricing financial assets. Numerous researchers have conducted research and tested different estimators of volatility. The volatility estimators have been improved as a result, and today's estimators are less noisy than standard deviation and will return clearer results (Daly, 2008; Engle, 1993; Molnár, 2012).

Volatility estimation, modeling, and forecasting can be considered some of the most developed parts of financial econometrics (Molnár, 2012). We study how attention to Elon Musk leads to Tesla's stock price variations. The variation will be estimated via a suitable volatility estimator (to be defined later). The focus will not be on returns as defined earlier but rather on the volatility connected to the returns. Some of the interests towards Elon Musk are positive and others negative, of which the former can lead to price increases, while the latter might lead to price declines. In this study, the content of the tweets will not be analyzed, and Google search volumes will also be used to measure the impact of attention on volatility, which measures whether prices change, irrespective of the direction of that change. The main focus is on Tesla, which, according to Nadeem (2021)'s examples, most times has been directly affected by Musk's tweets. Musk is also the CEO of the company, which could result in the research being applied to other public figures and CEOs of other major public companies.

2.3.1 Social media

Some articles and research have been done on how or if social media impacts the stock market and to what extent. This would be similar to this thesis, except we are looking at one person rather than social media as a platform, like Facebook or Twitter. Han and Yang (2013) explains how social networks, information acquisition, and asset prices, all three, are related in such a manner that any one of these can impact the other. In detail, that means that social networks affect asset prices and that social networks play a part in information acquisition. Social media and networks are great sources to find news and relevant information about companies or stocks as most companies today not only have public social media accounts but often the CEO and executive management within as well. By following not only the main account for the company but also executive management, you gain access to more news and statements that you otherwise would miss by just reading newspapers. More often than not, if there is larger attention and controversy surrounding social media postings, you will find the same in larger newspapers, but for smaller attention postings, there will not be any reports in newspapers. Another aspect of this is that the younger generation and youth today have a natural advantage as they already use social media several times a day.

According to Perrin (2015), already in 2015, nearly two-thirds of American adults use social networking sites daily. More specifically 65% of adults above the age of 18. Whereas the young adults, between the years of 18 and 29, almost 90% does. This can be considered expected, as young adults, especially on the lower end of the age scale, were youth when most technology and social networks were launched. There are some differences between the genders, where within the same group, 68% of women and 62% of men use social media daily. As expected, age is highly correlated with social media usage.

Pew Research Center began the surveys and tracking of social media activity back in 2005, a year after Facebook was launched. At that point, 12% of young adults (18-29), 8% of adults (30-49), 5% of adults (50-64), and 2% of seniors (65+) were using social media daily. In the 10-year time, to 2015, when the article was published, there was a 78%

increase for young adults, which is the largest growth in age, confirming the correlation between age and social media usage.

There are smaller differences in usage when looking at race, income, and education level. Most notable here is that from the highest-earning households (More than 75,000 USD per year) to the lowest-earning ones (less than 30,000 USD per year), there is a difference of 22% between the two, where 78% of the highest-earning ones use social media daily, compared to 56% of the lowest earning ones.

Based on the article and the numbers by the Pew Research Center, we can comfortably say that social media is a large part of people's daily lives. Even though the article and surveys capture U.S. citizens, it can like be closely compared to other Western countries. It also shows how social media gives not only people but companies the opportunity to reach followers all over the world. In terms of this thesis and research, we can reasonably assume that most people (above 50%) use social media daily and can be affected by postings done by Elon Musk for example. The results from this study give credibility to this approach as well. The results suggest that enough people are using the internet and social media for information retrieval, and many of those are likely (retail) investors. Moreover, as wealthier people are more likely to invest it is interesting to see that higher-earning households use social media even more.

2.3.2 Attention

People use their smartphones several times during the day, whether it is for reading the news, texting friends, or checking social media. The access to information about almost anything is at your fingertips. Individuals have limited attention or processing power to devote to one thing. In this setting, investments. One can either devote too much attention to irrelevant information or too little attention to important information (Barber & Odean, 2013; Hirshleifer & Teoh, 2003). Which information is relevant and important can be almost impossible to determine. Especially for retail investors, which not only applies to the limited attention hypothesis but is also limited by time. The limited attention hypothesis refers to how information about "everything" is available and that it is almost impossible to separate what information to gather and what not to do. Similarly, people tend to have limited time or attention span in order to gather all the information needed. This particularly applies to retail investors, as these ones typically have work and several other things to focus on, rather than information regarding the stock market (Hirshleifer & Teoh, 2003). That said, it also applies to investors or brokers who work in finance. Considering the vast amount of followers that Elon Musk has, it is natural to think that most of them are, in fact, retail investors, if investors at all, and that the hypothesis is applicable when researching areas like this. The research was done on attention and “doing-as-others” behavior is crucial to understand if there are grounds for investigating that behavior could affect the market in terms of trading volume, either buy- or sell orders, and if the spike in trading volume is large enough to cause volatility in the stock price. Further, how could this attention be generated in such a manner that a larger group of people were to follow and place an order on the word of the “news-caster”? Here, different researchers have looked at the question in different manners, e.g., Lyócsa et al. (2022), which tells the story of how the public went against larger investment banks who shorted the GameStop (GME) stock. Some retail investors caught on, and rather than join in on the short, they took to social media and gathered a lot of attention to this worldwide. Then, the retail investors grouped up and bought long positions into GameStop, which led to the stock increasing greatly rather than decreasing, as the investment banks had predicted. Lyócsa et al. (2022) called this riding with the herd, and this phenomenon, that people follow trends and copy others, will become relevant for this study, as this GameStop incident shows us, that if enough attention is gained and enough people buy the stock, it can impact the stock-price in an unexpected manner. The goal of this thesis is to determine if a celebrity or public figure can affect the equity market in terms of either being in a position of power, such as president (Gjerstad et al., 2021), senator, or similar, or through the fact that you can gather enough attention from the public that people will start buying or selling the financial instrument based on your word. Similar to Lyócsa et al. (2022), Nam and Skillicorn (2023) researched from the same angle, but were not looking at one specific example. On the other hand, we have articles that try and explain how forums and social media affect people and how they invest (Jiao et al., 2020; Ozsoylev & Walden, 2005; Piñeiro-Chousa et al., 2017; Sun et al., 2020). Another aspect of this is how people follow trends and how that either can strengthen or lessen the effect of economic impact, e.g., how at the start of economic crises (Shiller, 1992), people do as others, which strengthens the effect, and what makes it a crisis. As an example, the sub-prime mortgage scandal in the US probably would not have resulted in a worldwide financial crisis, if fewer people joined in and got the mortgages. As that was not the case, we cannot be certain that it would make a difference, but it puts things into perspective, such as how trends can strengthen effects. For this research, this is also something that we will be able to interpret if there is a significant effect on the public in terms of what Musk publishes on social media.

2.3.3 Elon Musks effect in other markets

The most researched area, with a similar theme as this thesis, is how Elon Musk can affect cryptocurrency value and the volatility to specific currencies, e.g. Ante (2023) and Huynh (2023). Even though this study is about the equity market, the research is similar in terms of both using the same models and data (tweets, re-tweets, favorites, etc). In the articles, Ante (2023) found significant positive abnormal returns and trading volume, and that this effect differs between different cryptocurrencies (*Bitcoin* and *Dogecoin*). On the other hand, Huynh (2023) concludes that the effect of Elon Musk's attitude could drive *Bitcoin* returns on short trading days, they were not able to find any supporting evidence for the impact of Musk's Twitter feed on *Bitcoin* volatility. As mentioned in the introduction, the thesis was written on inspiration from the Reuters article by Nadeem, 2021, which can also be compared to the articles related to the cryptocurrency market, since both clearly point to singular events when either the stock or value of cryptocurrency either skyrocketed or dropped. This gives the thesis incentive to find out if there is significant evidence that Musk can interfere with the stock price.

2.3.4 Measurements

Throughout the years, a lot of different research has been done concerning volatility models and measures, e.g. Daly (2008) and Molnár (2012). The main goal of these articles is to present and explain different volatility models and measures. Lyócsa et al. (2021) has tested time-related types of data and researched when to implement low- and high-frequency data. The current state-of-the-art approach is to use high-frequency data, but from Lyócsa et al. (2021), you can find that such data have limitations. High-frequency data is rarely available, or only available for shorter periods. At the same time, the processing of such data is more complicated than low-frequency data.

Lyócsa et al. (2021) shows that one can apply low-frequency estimators for the volatility models we use; although they appear to be less accurate, they capture well-known features of volatility sufficiently enough. The study argues that if the forecasting horizon increases, the difference between high- and low-frequency volatility estimators diminishes. Even though we are considering short-term day ahead predictions, where high-frequency estimators are more accurate, we argue that the low-frequency estimators are still useful, despite them being more noisy than the high-frequency estimators. That said, using open, close, high, and low data for the stock data in question, the volatility during the days will be observed, even though it will not be measured several times within the day.

2.3.5 Contribution to existing literature

This study aims to contribute to the existing literature in three ways. First, the attention between tweets of Elon Musk and interest via Google search volumes is related. Secondly, the attention to Tesla is related to Tesla's price variation, but not attention to Elon Musk. Lastly, it provide evidence that Google search volumes in general is a more useful leading indicator of Tesla's price variation, rather than the tweets from Elon Musk. These results are new and contribute to the understanding of how attention drive stock price changes, which has implications for investors and regulators who oversee the activities of person of public interest.

3 Methodology

3.1 The scientific method

To elaborate on the research process, one could start by looking at the ancient Greek philosophers first. Most of our understanding of what science is can be traced back from Aristotle all the way up to the "more modern times", with characters like John Stuart Mill, Karl Popper, and Thomas Kuhn as "the last major contributors" to the philosophy part of the scientific method. There are a vast number of important philosophers, scholars, and scientists not mentioned here. However, the focus here is to briefly illustrate the paradigm of our understanding of the scientific method therefore, only a few will be mentioned (Hepburn & Andersen, 2021).

Aristotle was the first well-established person who outlined what would become the inductive-deductive method. His works and ideas build upon others again, but this has to suffice for the review portion of the philosophy for this thesis. This method is what is known today as the pillar of the scientific method (Hepburn & Andersen, 2021).

Ethics and law in research

The ethics portion of this quick introduction to the topic is largely based upon the ideas from the same individuals as for the philosophy portion, with the addition of Emanuel Kant.

The focus on ethics within the philosophy of science became a more known and established fact after the scientific revolution in the 17th century (Ruyter, 2019). The ideas on which the different views on ethics have come from the foundation for the scientific methods used today, the philosophy of ethics within science. This has led to some ethical standards for researchers and laws once must oblige to.

Researchers are obliged to follow said ethical standards; this is both a requirement through laws and ethical standards. The laws that are particularly of interest are the GDRP, researcher laws. For researchers based in Norway, the guidelines given by the National Research Ethics Committees are the foundation for the ethics for research ("The National Research Ethics Committees", n.d.). They have listed many different guidelines for ethics, but not all of the guidelines are a concern for the work with this thesis. The guidelines for

Research Ethics in the Social Sciences and the Humanities and the A Guide to Internet Research Ethics are.

These guidelines are even rooted in Norwegian law; the forskningsetikkloven is clear that all research in Norway is to be conducted in a manner that upholds the acknowledged research ethical norms ("Lov om organisering av forskningsetisk arbeid (forskningsetikkloven) - Lovdata", 2017).

This can lead to a bit of a moral dilemma, as things that are legal are not necessarily morally right. In this work, we will strive to adhere to the laws and research ethics given by the governing bodies of the different legislation.

The practical impact of this for the thesis is that it follows the scientific method, adheres to different laws and ethical standards, and is aware that the findings of our work are only valid within the given scope of the current paradigm of knowledge.

3.2 Choice of research design

This thesis will be based on a quantitative method. However, we acknowledge that our data, research design, data exploration, and how we intercept those can be influenced by our personal biases.

Bias in research can come from many different sources, and researchers should do what they can to avoid it. A sound research design is a way to avoid this. It is a template for conducting the research so that one can try to mediate the risks as best as possible before one has to decide where to go, what to do, and how to interpret the data.

This thesis uses both a research-driven approach and a data-driven approach. First, it is based on whether or not Elon Musk's Twitter usage does, in fact, influence the volatility in Tesla's stock price.

3.3 Choice of data collection and analysis method

The methods used in this study are ordinary least squares (OLS), volatility models, HAR models, robust coefficients, and range-based estimators for volatility. Further into the methodology section, these will be explained, as well as how they were applied to the study. For this study, we need data from Twitter, Google search volume, stock data for the Tesla ticker, and VIX (Volatility index).

Firstly, the Twitter data (tweets from Elon Musk) was collected through Kaggle, which is a public repository of data sets. It is used by people all over the world, ranging from students to practitioners. Here, before Twitter implemented the payment solution for downloading tweets and data, one had already downloaded tweets from 2011 to 2021, which is the Twitter data used for this study.

Then, for the same period, Google search volume data for two search terms, "Elon Musk" and "Tesla", was downloaded directly from Google. This was done through R and code to make the process as automatic as possible. Google search volume is a separate service from Google, where you can find information regarding search terms and how many and when the search terms were searched for.

Stock data for the Tesla stock was collected through R, similar to the Google Search volume data. Using a package in R called "Quantmod", we collected the stock-data, for the same period as the tweet-data.

After the data was collected, it was cleaned and processed into one useful dataset. From here, the analyses began, with tests conducted to determine that the data could be applied to the models and that it was fine in terms of the assumptions for time series, as well as the HAR model. More on these later.

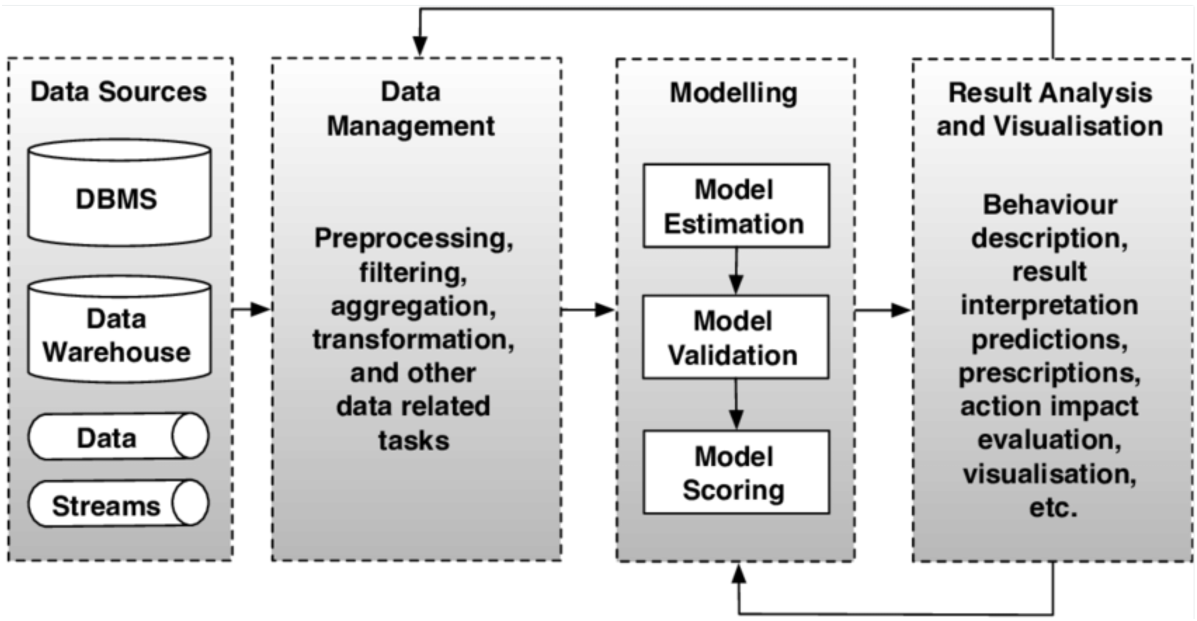
At this point, we wanted to include another variable for the models, called VIX, which is the CBOE (Chicago Board Option Exchange) volatility index for the U.S. stock-market, more concretely the 500 largest companies registered on the U.S. market. This is a popular estimation of expected volatility.

The ideas to use the range-based volatility estimators come from Molnár, 2012. Where Garman and Klass (1980), Rogers and Satchell (1991), and Parkinson (1980) were implemented. These are range-based volatility estimators and estimate the volatility in the stock price. The range refers to the difference between high and low prices, which is a natural candidate for the volatility estimation (Molnár, 2012). Later in this chapter, equations, and implementation for these estimators will be further abbreviated.

3.4 The data process

The data process of this thesis was quite vast and a challenge for the researchers. Here is a quick outline of what was done before diving a bit deeper into the different parts in the following chapters.

Assuncao et al. (2013) describes how workflows can be for big data. As the approach and how things were done regarding the data process, it is vital to be able to achieve the goals of this analysis. This workflow follows more or less a tidy approach to data collection Wickham (2014), but in a more general sense.



Assuncao et al. (2013)

This is in line with what was done, as the typical R packages for data transformation were not used. However, due to how comprehensive parts of it were, this will be presented in a slightly different manner.

Data collection: The approach was first to find the data needed for the analysis in line with that (Assuncao et al., 2013) showcases in the figure as Data Sources. The data was found at Kaggle, Google, and through an R package.

Data cleaning: This belonged to what (Assuncao et al., 2013) had in Data Management. The Twitter data and the Google search volume data were particularly messy. The data for the ticker symbols were more or less in line with what was desired.

Data aggregating: This part would also fall into what (Assuncao et al., 2013) had in Data Management. In this part of the work, the data was aggregated again, especially the Twitter and Google search volume data that needed work.

Data modeling: We based our models on the HAR model, but we used range-based volatility estimators (Molnár, 2012). This was done to see if the time series of volatility itself, with the addition of the Google search volumes and the Twitter usage of Elon Musk, influenced the volatility of the Tesla stock.

The results and visualizations are presented in chapter 4.

Data collection

As mentioned in the introductory part of this chapter, the data was collected from different sources, including Kaggle(for the Twitter data), Google, and Yahoo Finance.

As mentioned earlier, the Twitter data came from Kaggle due to Twitter's paywall. The publicly available data ensures that the results are replicable and testable for others. This also ensures that we do not go into the scope of the GDPR or the US equivalent since all the data was made public by the author himself. As such, they are to be considered public information. The Google Search volumes and stock data are also made publicly available and can be used without further permission.

Data cleaning

Data cleaning and tidying was a process that involved many steps.

Twitter data: the data spanned a long time period, with an NA value for a lot of the Twitter data. This is quite self-explanatory, as Elon Musk posted tweets on Twitter at all times, and even when he used it regularly, he still did not use it every day.

Here, the main challenge was to input NA values for the days that Elon did not use Twitter and to average out the usage over the weekends and other non-trading days.

Another challenge with the Twitter data was that there were days with multiple tweets, re-tweets, and so on. These all had to be aggregated onto the data as well. This was done in R using the mutate function, grouping the different records for each date and then storing the values. This was done for the tweets, likes, replies, and retweets.

Google search volume data: The main challenge here was that one could only download Google Search volumes for 90 days before getting data based on weeks instead of days. As we needed daily data for the analysis, the solution was downloading and combining many small data frames. This led to an operation in which the daily data was normalized with overlaps over the time frames so that every data frame downloaded had to overlap the last time frame and then be normalized when combined into a larger time series.

Symbols data: The symbols for Tesla (TSLA) and The Chicago Board Options Exchange (CBOE) Volatility Index (VIX). They were downloaded through Quantmod in R, and there were no real issues with them. The downloads were tidy, containing dates, and in a manner that was usable; they just needed to be appended to the existing data frame.

Data aggregating

As a starting point the symbol data for Tesla to start building our tidy data frame. On to this, we merged the data for the data for the VIX symbol. Then we added the Twitter data and the Google search volume data.

3.5 Data modelling

The first thing that was done was to see if Twitter use had an impact on stock prices. It showed significant results when utilizing a simple Ordinary Least Square model, with the returns or any of the volatility estimators as the dependent variable and only the number of tweets as the independent variable.

However, this changed when we moved to the HAR models. Then, the models showed that Twitter usage was not significant for the regression. With the Google search volumes added, the only significant of them was the Google search volumes for "Tesla".

This led to checking whether or not the Twitter usage of Elon Musk influenced Google search volumes. It did when running a model containing the Google search term for Tesla as the dependent variable and the number of tweets as the independent variable. It did show as significant. However, when we added the other Twitter variables, it showed that it was, in fact, not his use of Twitter but his engagement with other users in the form of likes and retweets that was significant.

The range-based volatility estimators used in this thesis were then implemented as the dependent variables. This was done by combining the GK and RS estimators as the dependent variable, leaving the PK estimator out of it to check whether or not the results were similar.

From this, the models were created. The first model was the standard HAR model. The second model was the HAR model with an average leverage added to it. The third model was the standard HAR model with the average leverage added to it and the log of the VIX squared.

The three different models were then added with the Google search volume data, and the Twitter activity was added for the last model.

3.6 Volatility estimators

In this section we define the range-based estimators of volatility that capture changes in asset prices, irrespective of their sign, i.e. irrespective of whether the price went up or down. To determine if there is or not, the variance or volatility is measured. Simply put, if there are any differences in the price on days that Elon Musk puts out tweets regarding Tesla, for example, compared to days where he does not. From the literature review section, the simplest intraday price variation will be:

$$\sigma^2 = (ln(C_t) - ln(O_t))^2 = IR_t^2 \tag{2}$$

But this estimator is known to be noisy, and a better approach would be to utilize intraday high and low prices. Let H_t and L_t denote highest and lowest prices and $H_t = ln(H_t)$ - $ln(O_t)$ and $L_t = ln(L_t)$ - $ln(O_t)$. The (Parkinson, 1980) estimator is given by:

$$PK_t = \frac{(h_t - l_t)^2}{4ln2} \tag{3}$$

And the (Garman & Klass, 1980) estimator which is even more efficient is given by:

$$GK_t = 0.511\,(h_t - l_t)^2 - 0.019\,(c_t(h_t + l_t) - 2h_t l_t) - 0.383c_t^2 \tag{4}$$

The (Rogers & Satchell, 1991) estimator:

$$RS_t = h_t(h_t - c_t) + l_t(l_t - c_t) \tag{5}$$

Where the notations in the equations is from prices open o_t, close c_t, high h_t, and low l_t. Next, the range-based volatility estimator used for this study:

$$RB_t = \frac{GK_t + RS_t}{2} \tag{6}$$

This is an average of the Rogers and Satchell (1991) and Garman and Klass (1980) and will be the range-based estimator for this study. The idea of using the average between the estimators came from Patton and Sheppard (2009). In this study, we only use the average of the two aforementioned.

Asymetric volatility

As discussed by (Horpestad et al., 2019), this thesis utilizes leverage measurements to test for asymetric volatility. The idea behind implementing this particular approach is to address the noticeable trend wherein negative returns often result in an increase in volatility. This trend has been observed in numerous instances, so it becomes essential to incorporate that and understand its implications.

This is done by calculating whether there is a negative intraday return for the Tesla stock. If there is, we multiply the logarithm of the estimator in question by 1 and use that as our leverage estimator or the value of the term is negative.

The equation for creating the leverage for the range-based (RB) estimator:

$$\text{RB\_leverage} \begin{cases} \ln(RB_t) \times 1 & \text{if IR} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

3.7 HAR model

When conducting analyses with time-series data, there are assumptions about the time-series that one should check for. Different literature defines and explains these in more or fewer assumptions, where they differ between four to six assumptions about time-series (Hair Jr et al., 2019; Kennedy, 2008; Wooldridge, 2020). In short, they are linearity, collinearity, homoscedasticity, normality, and no serial correlation. That is, it is summarized and collectively mentioned within the references. The model of choice is the heterogeneous autoregressive (HAR) model of Corsi (2009) that has been used in many empirical applications (i.e. Lyócsa et al. (2021), Woebbeking (2021), and Patton and Sheppard (2009)). The model is estimated in an ordinary least squares (OLS) framework. Before we write the considered specifications, let us briefly describe the OLS model first. In accordance with Wooldridge (2020), the ideal assumptions of a time-series and OLS model are as follows:

Linearity

"The stochastic process $\{ (x_{t1}, x_{t2}, ..., x_{tk}, y_t)$: $t = 1, 2, ..., n \}$ follows the linear model

$$Y_t = \beta_0 + \beta_1 X_{t1} + ... + \beta_k X_{tk} + u_t, \tag{8}$$

Where $\{ u_t$: $t = 1, 2, ..., n \}$ is the sequence of errors or disturbances. Here, n is the number of observations (time periods)."

No perfect collinearity

"In the sample (and therefore in the underlying time series process), no independent variable is constant nor a perfect linear combination of the others."

Homoscedasticity

"Conditional on X, the variance of u_t is the same for all t: $\text{Var}(u_t \mid \mathbf{X}) = \text{Var}(u_t) = \sigma^2$, $t = 1, 2, ..., n$."

Normality

"The errors u_t are independent of $\mathbf{X}$ and are independently and identically distributed as Normal$(0, \sigma^2)$"

No serial correlation

"Conditional on $\mathbf{X}$, the errors in two different time periods are uncorrelated: $\mathrm{Corr}(u_t, u_s \mid \mathbf{X}) = 0$, for all $t \neq s$."

All of this has been tested for the data, by different tests that are either mentioned or will be explained later in this chapter.

Today, the availability and access to high-frequency intraday data have made newer literature focus on employing realized variance (RV) to build forecasting models (Clements & Preve, 2021). The HAR-model was introduced by Corsi (2009) and gained popularity due to its simplicity both in terms of understanding and implementation. From the paper Corsi (2009), the original HAR-model specifies RV as:

$$RV_t = \beta_0 + \beta_1 RV_{t-1}^d + \beta_2 RV_{t-1}^w + \beta_3 RV_{t-1}^m + \mu_t \tag{9}$$

Where the β_j $(j = 0, 1, 2, 3)$ is an unknown variable that will be estimated, and the RV_t is the realized variance for day t. RV_{t-1}^d, RV_{t-1}^w, and RV_{t-1}^m denote the daily, weekly, and monthly lagged RV, and μ_t is the error term.

Equation (9), the HAR model, is similar to linear regression, which can explain how many find the model easily interpreted and implemented, and why it has become commonly used for modeling realized volatility.

The range-based volatility estimator that has been used in this study is, as aforementioned, an average of the Garman and Klass (1980) and Rogers and Satchell (1991). This means that the model used for this study is a HAR-RB (HAR - Range Based) model, defined as:

$$RB_t = \beta_0 + \beta_1 RB_{t-1}^d + \beta_2 RB_{t-1}^w + \beta_3 RB_{t-1}^m + \mu_t \tag{10}$$

Where the notations are equal to the ones in equation (9). Building on this and including the logarithmic form for the variables, in order to stay in line with the assumptions, we present the first three models, which will be referred to as the benchmark models.

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) \tag{11}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(leverage_{t-1}) + \mu_t \tag{12}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(leverage_{t-1}) + \beta_5 ln(VIX_{t-1}) + \mu_t \tag{13}$$

Equation (11) is the HAR-RB model, presented in equation (10), but with the logarithms of the variables. Equation (12) is the HAR-RB model with the *leverage* variable added, and equation (13) is the HAR-RB model with *leverage* and *VIX* variables. These three models (Eq. 11-13) can be found in table 5 and will be referred to as the benchmark models for this study.

Moving on, testing the impact that Google search volume and Twitter variables on the benchmark models:

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(GTT_t) + \beta_5 ln(GTEM_t) + \mu_t \tag{14}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(Leverage_{t-1}) + \beta_5 ln(GTT_t) + \beta_6 ln(GTEM_t) + \mu_t \tag{15}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(Leverage_{t-1}) + \beta_5 ln(VIX_{t-1}) + \beta_6 ln(GTT_t) + \beta_7 ln(GTEM_t) + \mu_t \tag{16}$$

This is the HAR-RB models, including *leverage* and *VIX* plus the Google search volume variables. The latter variables are GTT, which is the Google Search volume for "Tesla", and $GTEM$ is the Google Search volume for "Elon Musk". These models are presented in Table 6 under the results section of this thesis.

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(tweet_t) + \beta_5 ln(retweet_t) + \beta_6 ln(reply_t) + \beta_7 ln(likes_t) + \mu_t \tag{17}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(Leverage_{t-1}) + \beta_5 ln(tweet_t) + \beta_6 ln(retweet_t) + \beta_7 ln(reply_t) + \beta_8 ln(likes_t) + \mu_t \tag{18}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(Leverage_{t-1}) + \beta_5 ln(VIX_{t-1}) + \beta_6 ln(tweet_t) + \beta_7 ln(retweet_t) + \beta_8 ln(reply_t) + \beta_9 ln(likes_t) + \mu_t \tag{19}$$

Similarly, the HAR-RB models plus the Twitter variables *tweet*, *retweet*, *reply*, and *likes*. Presented in table 7. Final tests are models that include every variable, both Twitter and Google search volumes related.

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(GTT_t) + \beta_5 ln(GTEM_t) + \beta_6 ln(tweet_t) + \beta_7 ln(retweet_t) + \beta_8 ln(reply_t) + \beta_9 ln(likes_t) + \mu_t \tag{20}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(Leverage_{t-1}) + \beta_5 ln(GTT_t) + \beta_6 ln(GTEM_t) + \beta_7 ln(tweet_t) + \beta_8 ln(retweet_t) + \beta_9 ln(reply_t) + \beta_{10} ln(likes_t) + \mu_t \tag{21}$$

$$ln(RB_t) = \beta_0 + \beta_1 ln(RB^d_{t-1}) + \beta_2 ln(RB^w_{t-1}) + \beta_3 ln(RB^m_{t-1}) + \beta_4 ln(Leverage_{t-1}) + \beta_5 ln(VIX_{t-1}) + \beta_6 ln(GTT_t) + \beta_7 ln(GTEM_t) + \beta_8 ln(tweet_t) + \beta_9 ln(retweet_t) + \beta_{10} ln(reply_t) + \beta_{11} ln(likes_t) + \mu_t \tag{22}$$

These models are presented in tables 5 to 8 under results, with the rest of the models above.

3.8 Model selection

When comparing multiple models, we can use discrete measures to compare the models up against each other, such as R^2, adjusted R^2, Akaike's information criterion (AIC), and Bayesian information criterion (BIC). Goodness-of-fit (GoF) is a term used to describe to which extent the models fit the data. R^2 and adjusted R^2 explains the proportion of variance in the dependent variable explained by the model. Augmented Dickey-Fuller, Escanciano-Lobato, Breusch-Pagan, and Shapiro-Wilk are other tests used to test the models and time series of interest. Where these tests are designed to check for different things. The augmented dickey-fuller (ADF) test was used to test for unit root and stationarity. ADF tests whether the time series has a unit root, which would imply a specific form of non-stationarity. (Dickey & Fuller, 1979). Escanciano-Lobato, the automatic Portmanteau, is testing for serial correlation in a given time series (variable of interest or residuals from a model)(Escanciano & Lobato, 2009). Breusch-Pagan tests for heteroskedasticity of residuals from a model (Breusch & Pagan, 1980). Shapiro-Wilk is a test for normality, i.e. if the variables of interest can be assumed to follow a normal distribution (Shapiro & Wilk, 1965).

Bayesian information criterion (BIC) and Akaike's information criterion (AIC) are measurements that are especially useful when comparing models, and they also take the under- and over-fitting of models into consideration. In other words, it considers the trade-off between more advanced models against the risk of over-fitting (Aho et al., 2014). AIC and BIC are defined as (Aho et al., 2014):

$$AIC = -2ln \times L(\hat{\theta}) + 2 \times p \tag{23}$$

$$BIC = -2ln \times L(\hat{\theta}) + p \times ln \times n \tag{24}$$

Where $L(\hat{\theta})$ is the value of the likelihood function, p is the total number of parameters included in the model, and lastly n is the sample size (Aho et al., 2014). From these measures, including R^2 and adjusted R^2, the models are tested up against the data and tested for under- and over-fitting. To find the best model, baseline benchmark models will first be established and then built on top of these in terms of variables.

Tests like Dickey-Fuller (1979), Escanciano-Lobato (2009), Breucsch-Pagan (1980), and

Shapiro-Wilk (1965), will be conducted and used as presented in the original papers. This will be applied so that we can be confident that the data and models are as good as they can be. For the Dickey-Fuller test specifically, the augmented version will be used.

The testing for serial correlation was done with a portmanteau test. A robustified portmanteau test with automatic lag selection was applied Escanciano and Lobato, 2009, this was done with the Auto.Q function in the r package vrtest. This test returns p-values for the models and will effectively help determine if the model captures the persistence of the volatility

3.9 Time series and assumptions

Time series data is typically described as data with temporal ordering and utilizes non-random sampling. In other words, the data and variables follow a chronological order and with equal time differences (Wooldridge, 2020). Similar to Wooldridge (2020), Brockwell and Davis (1991) says: "a time series is a set of observations x_t, each one being recorded at a specified time t". The latter can be interpreted as a definition of time series. For this study, stock-market data is used and is chronologically ordered for dates where the non-trading days (weekends, holidays, etc.) have been removed so it is possible to do our research properly. Similarly, the tweet data for Elon Musk is ordered on the same dates, whereas his weekend tweets have been pushed back to the last trading day. Friday for weekends, and the day before holidays. As this thesis utilizes time series for its data, certain aspects must be considered. Wooldridge (2020) does not present a real definition, but rather what is considered a time series.

The methods are the same as for OLS but somewhat different. For OLS, the data cannot be correlated with the dependent variable. This is almost impossible when dealing with time series; the lagged versions of the time series will almost always have some memory of their past values.

4 Results

Moving on from the methodology, and how the study was conducted. The study is data-driven, which leaves it natural to test several models and the data, before defining the final models. Here, the final models will be presented, and the key points of each of the models. In chapter 5 Discussions, the results, and models will be further interpreted and discussed.

4.1 Descriptive statistics

The data was a data frame, comprising aggregated, combined, calculated, and imported data from many different time series. The starting date of this data was 23.11.2011 and the end date was 16.04.2021. It contained Date, Open, High, Low, Close, Volume, and Adjusted Close for the Tesla stock, as well as VIX. From the attention measures, it contains the number of daily tweets, retweets, replies, likes, and the Google search volume for "Tesla" and "Elon Musk." From this, we calculated, the Intraday Returns (IR), Overnight Returns (OR), Jump component (JC), Rogers and Satchell (RS), Garman and Klass (GK), and Parkinson (PK) calculated using "c", "h", and "l" components showcased by Molnár, 2012. As well as lagged variables of some of the different variables. The variables directly used in the data analysis are shown in Table 1: Descriptive statistics for key variables.

Table 1: Descriptive statistics for key variables

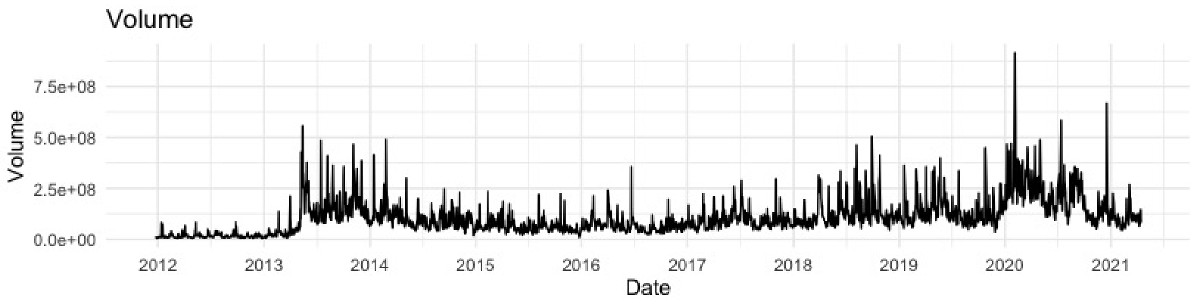
	mean	sd	min	max	skew	kurtosis	$\rho(1)$	$\rho(5)$	$\rho(22)$	EL
Rogers and Satchell - RS_t	1963.90	3425.89	0.00	45888.25	6.79	62.21	0.43	0.21	0.09	***
Garman and Klass - GK_t	1963.72	3274.28	80.56	50737.68	6.71	63.81	0.46	0.21	0.10	***
Parkinson - PK_t	1957.48	3288.43	74.77	48863.65	6.33	58.65	0.41	0.19	0.08	***
Range-based estimator - RB_t	1963.81	3325.04	60.07	48312.97	6.73	62.39	0.45	0.22	0.09	***
Daily tweets	3.69	5.92	0.00	48.00	2.69	9.78	0.41	0.34	.031	***
Daily retweets	8341.02	27678.56	0.00	597082.00	8.78	122.50	0.20	0.21	0.11	***
Daily replys	2531.15	8169.10	0.00	149365.00	7.45	80.34	0.34	0.36	.024	***
Daily likes	78980.12	238215.79	0.00	4821004.00	7.68	99.53	0.32	0.35	0.24	***
Google Search Volume "Elon Musk" ($GTEM_t$)	26.69	56.33	0.08	1414.80	14.85	300.27	0.86	0.40	0.20	***
Google Search Volume "Tesla" (GTT_t)	28.04	24.27	4.32	587.75	7.69	140.93	0.72	0.53	0.5	***
Intraday Returns - IR	0.02	2.78	-22.01	14.65	-0.02	4.20	-0.05	-0.01	0.01	**
Overnight Returns - OR	0.19	2.23	-16.13	22.86	0.35	16.38	0.02	0.02	0.01	*
Jump Component - JC	1263.54	5403.26	0.00	131705.70	11.42	191.31	0.10	0.12	0.02	***
VIX	16.97	7.01	9.14	82.69	3.39	18.81	0.96	0.87	0.56	***

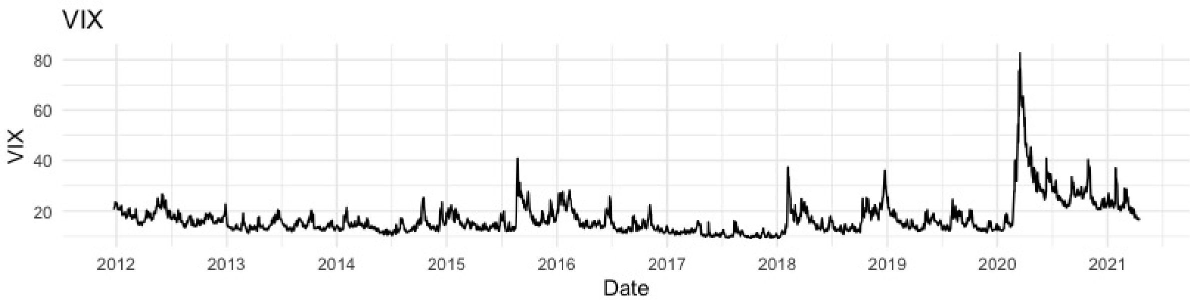
Note: ACF lag1 - $\rho(1)$, ACF lag2 - $\rho(5)$, ACF lag22 - $\rho(22)$, significance level from Escanciano and Lobato test - EL

The thesis comprises two different approaches to research design, both problem-driven and data-driven. After the data was collected with a basis in the problem-driven approach, we then tried to learn what we could from the data we had collected. We ended up with a data-driven approach. This can be viewed as getting to know our data, what we can learn from it, and utilizing it to gain knowledge on the different aspects of what the data tells.

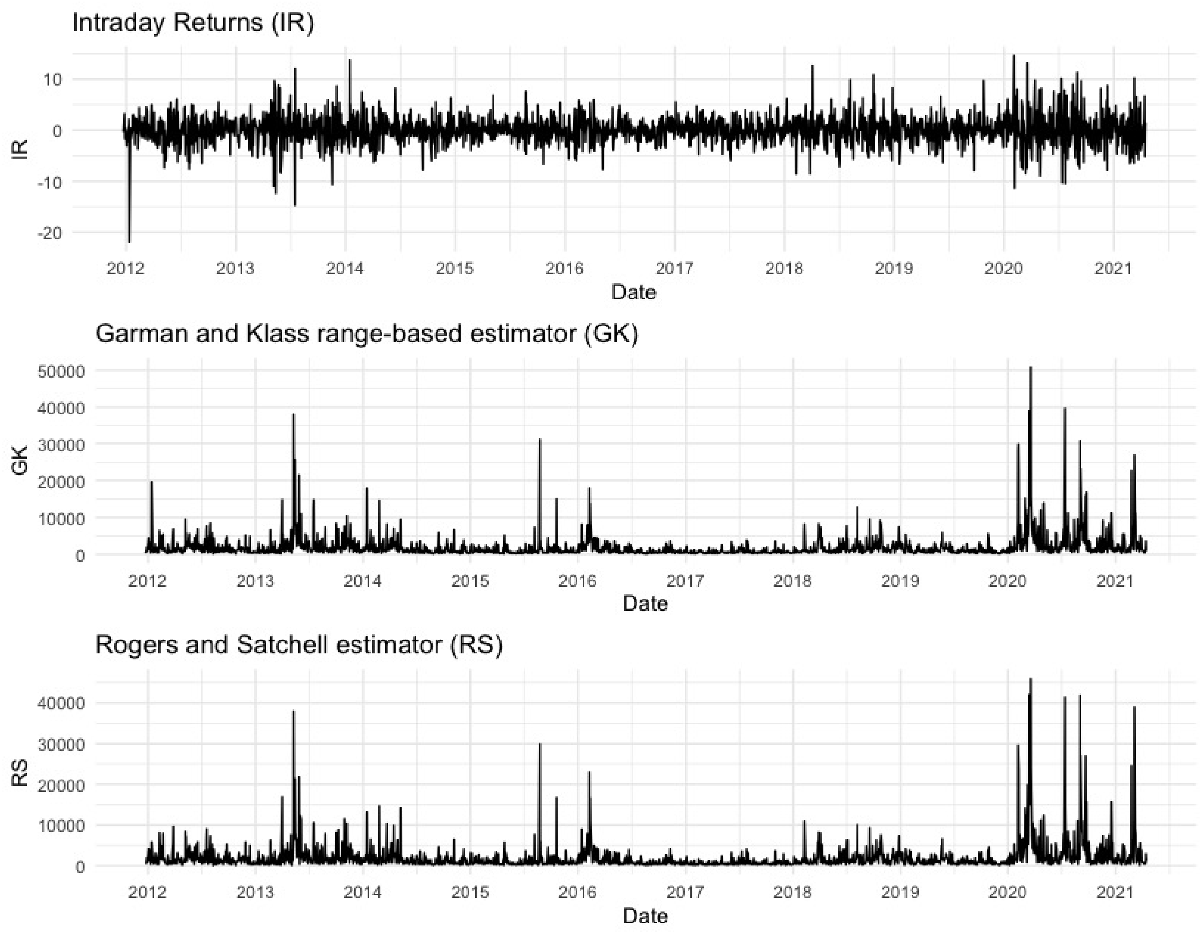
The research question was whether there is an Elon Musk effect from his use of Twitter and how it impacts the volatility of the Tesla stock. The first thing to check was whether future Google search volume intensity, related to Elon Musk and Tesla, was led by his Twitter activity, which we consider to be a natural starting point.

The adjusted closing price for Tesla stock, trade volume, and VIX are graphically shown below, providing a visual presentation of the data.





From the first graph, the key point is that Tesla's market value (Price times stocks in circulation) was even until early 2020 when the stock price skyrocketed. Even though it was not until early 2020 that the stock started rising, the volume traded has been relatively stable since mid-2013, with a spike around the same time the market value started rising, as can be seen in the second graph. The VIX shows the same as the volume, plus a smaller spike in late 2015. The VIX corresponds to the U.S. stock market and is an index explaining the volatility of stocks. Here, volatility for the U.S. stock market spiked in early 2020, which can be traced back to the COVID-19 pandemic, where most stocks considerably declined around the world, and not only in the U.S. market.



For the intraday-, and overnight returns, Parkinson's, Garman-Klass, and Rogers and Satchell the test returns p-values below 0.01. The time series presented here is considered stationary, and the spikes that can be seen in the graphs are not large and are not frequent often enough to consider the time series to have a unit root and thus be non-stationary.

The testing for serial correlation was done with a portmanteau test. A robustified portmanteau test with automatic lag selection was applied Escanciano and Lobato, 2009, this was done with the Auto.Q function in the r package vrtest. Results show that serial correlation was present in most of our data which supports our choice of auto-regressive data framework to model volatility, i.e. to use the HAR model that captures this persistence of the time-series.

Table 2: correlation matrix, Pearson (lower triangular) and Spearman (upper triangular)

	RS	GK	PK	tweet	retweet	reply	likes	GTEM	GTT	IR	OR	JC	VIX
RS	1	0.956	0.825	0.104	0.086	0.107	0.096	0.065	0.117	0.054	0.012	0.320	0.383
GK	0.970	1	0.946	0.114	0.095	0.117	0.106	0.084	0.144	0.042	0.015	0.340	0.388
PK	0.847	0.938	1	0.112	0.092	0.113	0.103	0.091	0.156	0.026	0.021	0.330	0.356
tweet	0.108	0.117	0.118	1	0.927	0.944	0.935	0.577	0.565	0.011	0.046	0.160	0.182
retweet	0.042	0.051	0.060	0.437	1	0.983	0.980	0.604	0.586	0.001	0.026	0.153	0.162
reply	0.100	0.115	0.120	0.485	0.808	1	0.992	0.663	0.647	0.002	0.032	0.174	0.199
likes	0.074	0.084	0.092	0.516	0.933	0.894	1	0.680	0.666	0.002	0.026	0.168	0.197
GTEM	0.087	0.088	0.084	0.332	0.259	0.282	0.305	1	0.931	0.006	0.014	0.209	0.172
GTT	0.225	0.231	0.222	0.386	0.326	0.401	0.409	0.370	1	0.004	0.033	0.255	0.143
IR	−0.069	0.056	−0.041	0.027	0.0003	0.009	0.004	0.002	0.019	1	−0.050	−0.009	-0.067
OR	0.047	0.054	0.047	0.058	0.004	-0.002	0.001	0.025	0.026	-0.020	1	0.162	0.025
JC	0.393	0.392	0.322	0.066	0.040	0.050	0.049	0.052	0.185	-0.034	0.120	1	0.273
VIX	0.418	0.417	0.371	0.255	0.186	0.249	0.243	0.277	0.254	-0.069	-0.031	0.217	1

Table 2 contains a correlation matrix, with correlations estimated by Pearson (lower triangular) and Spearman (upper triangular) methods. It shows that the three different range-based estimators utilized in the thesis are strongly correlated, which is to be expected as they estimate the same thing, with a slight difference in formulas.

The matrix shows that tweets correlate with retweets, replies, and likes of about 0.40 - 0.50 for the Pearson method but around 0.90 with Spearman. For the correlation between tweets and Google search volumes, Pearson is between 0.33 and 0.38, and Spearman is between 0.56 and 0.57.

The correlation between the average of the RB variable and the Google search volumes shows a correlation between 0.09 and 0.23 for Pearson and 0.08 - 0.13 for Spearman. This indicates that tweets are more correlated with the volatility estimator than Google search volumes.

4.2 Data exploration models

The models presented in tables (3) and (4), are the ones that we conducted to test the data and find relationships in line with the research questions. In table (3) we tested the lagged variables for the Google search volumes and the Twitter data.

Table 3: Models to check if tweets correlate with Google search volumes, and lagged versions if it self

	Dependent variable:	
	GTT_ave	GTEM_ave
	(1)	(2)
GTT_{t-1}	0.816***	
	(0.013)	
$GTEM_{t-1}$		0.853***
		(0.011)
$tweet_{t-1}$	0.159***	0.414***
	(0.050)	(0.127)
$retweet_{t-1}$	−0.00004*	0.0001*
	(0.00003)	(0.0001)
$reply_{t-1}$	0.0001	0.001***
	(0.0001)	(0.0002)
$likes_{t-1}$	0.00000	−0.00003***
	(0.00000)	(0.00001)
Constant	4.259***	2.366***
	(0.385)	(0.708)
Observations	2,341	2,341
R^2	0.732	0.752
Adjusted R^2	0.731	0.752
Residual Std. Error (df = 2335)	10.877	27.938
P-value from robustified portmanteau	0.51	0.52
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 3 shows an OLS regression using standard coefficients, which was done to test the impact of the lagged variables of the different attention metrics on the Google search volumes.

From this table and model (1), it is seen that for the Google search volume for "Tesla," the lagged variable itself, tweets, and retweets are significant. It has a value of 0.51 from the robustified portmanteau test, indicating that there is no significant autocorrelation in the model.

From model (2), it is seen that for the Google search volume for "Elon Musk," the lagged variable itself, tweets, retweets, replies, and likes, is significant. It has a value of 0.51 from the robustified portmanteau test, indicating that there is no significant autocorrelation in the model.

To sum up; there is a memory in the Google search volumes and for the Google search volume for "Elon Musk," there is a clear significant relationship that has coefficients that matter. For the Google search volume for "Tesla", the value of the significant variable is so low that we can not say clearly that this has an impact.

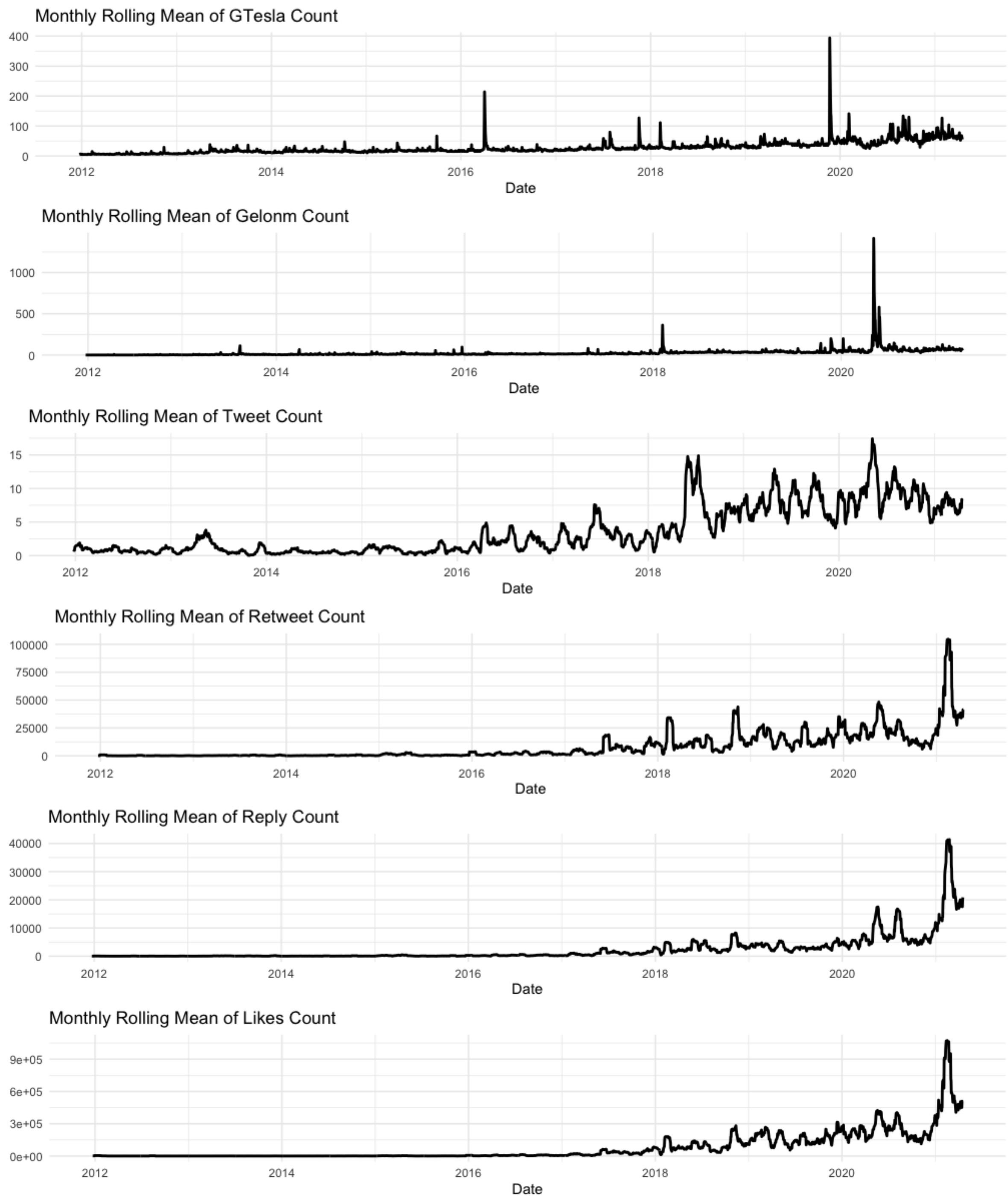
Table 4: Models to check what drives tweets, if there are correlations that could explain the Twitter activity

	Dependent variable:tweet_ave	
	tweet_ave	
	(1)	(2)
$tweet_{t-1}$	0.370***	0.310***
	(0.021)	(0.023)
$GTEM_{t-1}$	0.012***	0.011***
	(0.002)	(0.004)
GTT_{t-1}	0.072***	0.051***
	(0.006)	(0.009)
$retweet_{t-1}$	0.00001	0.00001
	(0.00001)	(0.00001)
$reply_{t-1}$	0.0001**	0.0001**
	(0.00003)	(0.00003)
$likes_{t-1}$	−0.00001***	−0.00001***
	(0.00000)	(0.00000)
$tweet_{t-2}$		0.157***
		(0.023)
$GTEM_{t-2}$		−0.002
		(0.004)
GTT_{t-2}		0.018**
		(0.009)
$retweet_{t-2}$		0.00001
		(0.00001)
$reply_{t-2}$		0.00001
		(0.00003)
$likes_{t-2}$		−0.00000
		(0.00000)
Constant	0.177	0.008
	(0.164)	(0.171)
Observations	2,341	2,340
R^2	0.307	0.324
Adjusted R^2	0.305	0.321
Residual Std. Error	4.643 (df = 2334)	4.592 (df = 2327)
P-value from robustified portmanteau	0	0.46
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 4 shows an OLS regression using standard coefficients, which was done to test the impact of the lagged variables of the different attention metrics on the number of daily tweets.

This table and model (1) show that there is autocorrelation due to its portmanteau test value of 0. Thus, there was autocorrelation, leading to the creation of model 2.

One key takeaway from this table was that we had to go to a model containing not just lag1 but lag2 before we had a model without autocorrelation in the residuals.



From this figure, the monthly mean with a rolling window of the Google search volume for "Tesla" and "Elon Musk", the number of tweets, retweets, replies, and likes can be observed.

It needs to be pointed out that the rolling mean of Google search volumes was expected to be below 100, as that is the maximum value that Google gives its search volumes. However, when the data was aggregated with a min-max function over a longer monthly time series from Google, some extreme spikes in the data came out. This was probably due to extremely high interest in either “Tesla” or “Elon Musk” at different times.

The figure shows that the Google search volume for “Tesla” has spiked around the launch time of some of its models and for “Elon Musk” at other times, around the time when he became the richest person in the world.

The other graphs show that his Twitter usage started to increase around 2016, with a slight lag in retweets, replies, and likes. This might be because he initially did not have the followers to get that many retweets, likes, and replies.

4.3 Benchmark models

Table 5: HAR (1), HAR + Leverage (2), HAR + Leverage + VIX (3)

	Dependent variable: $\ln(RB_{t+1})$		
	(1)	(2)	(3)
$\ln(RB_t^d)$	0.216***	0.215***	0.205***
	(0.028)	(0.028)	(0.028)
$\ln(RB_t^w)$	0.226***	0.221***	0.209***
	(0.044)	(0.044)	(0.045)
$\ln(RB_t^m)$	0.357***	0.359***	0.279***
	(0.038)	(0.038)	(0.043)
$ln(leverage_t)$		0.006	0.005
		(0.004)	(0.004)
$\ln(VIX_t^2)$			0.178***
			(0.039)
Constant	1.281***	1.287***	1.047***
	(0.199)	(0.198)	(0.214)
Observations	2,342	2,342	2,342
R^2	0.337	0.338	0.346
Adjusted R^2	0.336	0.336	0.345
Residual Std. Error	0.778 (df = 2338)	0.778 (df = 2337)	0.773 (df = 2336)
F Statistic	396.031*** (df = 3; 2338)	297.668*** (df = 4; 2337)	247.599***
AIC	5,476.135	5,476.086	5,446.528
BIC	5,504.929	5,510.638	5,486.840
P-value from robustified portmanteau	0.004	0.552	0.003

Note: *p<0.1; **p<0.05; ***p<0.01

Here are the three first models; moving on, these will be referred to as the benchmark models. The first model is a HAR-RB model; model 2 is the HAR-RB model, including leverage, and model 3 is the HAR-RB model, including both leverage and VIX. These models were presented under 3.7 section of this thesis (eq. 11-13).

These models are HAR-RB models consisting of logarithm to the variables RB_t^d, RB_t^w, and RB_t^m. The logarithm to the average of the Rogers and Satchell- and Garman and Klass estimator. The d, w, and m in the variables stands for daily, weekly, and monthly; that is, the variables are the daily, weekly, and monthly average of the RS- and GK-estimators. Where all the variables have a significant impact on the dependent variable, with a p-value of 0.01. This indicates that the variables within the model are strong, as opposed to higher p-values or not significant at all. R^2 and adjusted R^2 is 0.336 which means that the model explains close to 34% of the variation in the dependent variable. Based on adjusted R^2, AIC, and BIC, the preferred model out of these three is model (3), including both *Leverage* and *VIX*.

From the robustified portmanteau test, we can see that models 1 and 3 are significant, whereas model 2 is not, considering a significance level of 10%. These first models are both simplistic without including too many variables and establish the basis for further analyses. Moving on, tweets from Elon Musk, and Google search volumes for both Elon Musk and Tesla will be added to these benchmark models.

The only variable with a significant relationship with the dependent variable besides the lagged versions of the RB_t variables is the VIX (volatility index) variable has a positive impact of 0.178% on the dependent variable. Simply put, one unit increase in the VIX variable leads to a 0.178% increase in the dependent variable RB.

Table 6: Benchmark-models with Google search volumes

	Dependent variable: $\ln(RB_{t+1})$		
	(4)	(5)	(6)
$\ln(RB_t^d)$	0.210***	0.208***	0.200***
	(0.028)	(0.028)	(0.028)
$\ln(RB_t^w)$	0.219***	0.214***	0.203***
	(0.044)	(0.044)	(0.045)
$\ln(RB_t^m)$	0.353***	0.355***	0.282***
	(0.040)	(0.040)	(0.044)
$ln(leverage_t)$		0.007	0.005
		(0.004)	(0.004)
$\ln(VIX^2)$			0.174***
			(0.040)
GTT_t	0.002*	0.002*	0.002*
	(0.001)	(0.001)	(0.001)
$GTEM_t$	−0.0001	−0.0001	−0.0004**
	(0.0002)	(0.0002)	(0.0002)
Constant	1.338***	1.345***	1.075***
	(0.212)	(0.211)	(0.222)
Observations	2,342	2,342	2,342
R^2	0.340	0.341	0.349
Adjusted R^2	0.339	0.339	0.347
Residual Std. Error	0.776 (df = 2336)	0.776 (df = 2335)	0.772 (df = 2334)
F Statistic	240.719*** (df = 5; 2336)	201.067*** (df = 6; 2335)	178.458*** (df = 7; 2334)
AIC	5,469.180	5,468.982	5,442.496
BIC	5,509.491	5,515.052	5,494.325
P-value from robustified portmanteau	0.003	0.591	0.004

Note: *p<0.1; **p<0.05; ***p<0.01

Three more models, benchmark models + the Google trend variables, for the search terms "Elon Musk" and "Tesla". The table is presented as GTT_t for Tesla and $GTEM_t$ for Elon Musk. The variables from the benchmark model have some indistinguishable changes and remain statistically significant, as expected. These models is presented in equations 14-16.

In model 4, the Google trend variable for Tesla is significant with a p-value of less than 0.1, while the Google trend variable for Elon Musk is insignificant. Model 5 presents the same findings as model 4. In model 6, the Google trend for Tesla remains the same, but the Elon Musk Google trend variable also comes out as significant with a p-value of less than 0.05.

From the coefficient GTT_t (Google search volume for "Tesla"), we can see that one unit increase in "attention" (GT search volume) will result in a 0.2% increase in volatility. This is applicable to all the models in Table 6. In model (6), there is also a significant coefficient for $GTEM_t$ (Google search volume for "Elon Musk"). That means that one unit increase in attention results in a 0.04% decrease in volatility.

R^2 and the adjusted R^2 display the same values as previous models, 34%, and above, where again the model with the logarithm of VIX-variable displays the highest results of 0.349 (R^2) and 0.347 (Adj. R^2). For these models, the AIC is slightly changed downwards, with values of 5 469.18, 5 468.92, and 5 442.50, while the BIC is changed slightly upwards, with values of 5 509.49, 5 515.05, and 5 494.33.

Lastly, the p-values of the robustified portmanteau test return 0.003, 0.591, and 0.004, where again, the first and last model in the table (4 and 6) returns significantly. Model 5 does not.

Compared to the benchmark models in Table 5, we can see that the adjusted R^2 has shifted upwards, while AIC and BIC have shifted slightly downwards. Which means, that the latter models (4-6) are improved compared to the benchmark models.

Table 7: Benchmark models with tweet variables

	Dependent variable: $\ln(RB_{t+1})$		
	(7)	(8)	(9)
$\ln RB_t)$	0.214***	0.213***	0.204***
	(0.028)	(0.028)	(0.028)
$\ln(RB_t\_W)$	0.224***	0.219***	0.209***
	(0.044)	(0.044)	(0.045)
$\ln(RB_t\_M)$	0.348***	0.350***	0.281***
	(0.039)	(0.039)	(0.043)
$ln(leverage_t)$		0.007	0.005
		(0.004)	(0.004)
ln(VIXˆ2)			0.165***
			(0.039)
tweet	0.002	0.002	0.001
	(0.003)	(0.003)	(0.003)
retweet	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
reply	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
likes	0.000	0.000	−0.000
	(0.000)	(0.000)	(0.000)
Constant	1.351***	1.357***	1.094***
	(0.207)	(0.206)	(0.223)
Observations	2,342	2,342	2,342
R^2	0.340	0.340	0.348
Adjusted R^2	0.338	0.338	0.345
Residual Std. Error	0.777 (df = 2334)	0.777 (df = 2333)	0.773 (df = 2332)
F Statistic	171.660*** (df = 7; 2334)	150.554*** (df = 8; 2333)	138.115***
AIC	5,473.804	5,473.604	5,449.832
BIC	5,525.633	5,531.191	5,513.179
P-value from robustified portmanteau	0.003	0.498	0.003

Note: *p<0.1; **p<0.05; ***p<0.01

Moving on to tweets, the models in Table 7 include the tweet variables from the dataset. These variables are the tweets from Elon Musk himself, retweets, replies to the original tweet, and likes. None of the models present a significant result for the new variables, and the original ones have no remarkable change. Equations 17-19 are these models, under 3.7.

In terms of the adjusted R^2, AIC, and BIC, the adjusted R^2 has again shifted upwards, which means these models explain more of the dependent variable than the benchmark models do. AIC and BIC on the other hand, tell us that the benchmark models are better (AIC/BIC has increased).

Table 8: Benchmark-models with Google search volumes and tweet variables

	Dependent variable: $\ln(RB_{t+1})$		
	(10)	(11)	(12)
$\ln(RB_t)$	0.210^{***}	0.208^{***}	0.200^{***}
	(0.028)	(0.028)	(0.028)
$\ln(RB_t\_W)$	0.220^{***}	0.215^{***}	0.204^{***}
	(0.044)	(0.044)	(0.045)
$\ln(RB_t\_M)$	0.351^{***}	0.353^{***}	0.285^{***}
	(0.040)	(0.040)	(0.044)
$ln(leverage_t)$		0.007	0.005
		(0.004)	(0.004)
ln(VIX^2)			0.170^{***}
			(0.041)
GTT_t	0.002	0.002	0.002
	(0.001)	(0.001)	(0.001)
$GTEM_t$	−0.000	−0.000	-0.001^{***}
	(0.000)	(0.000)	(0.000)
tweet	0.001	0.001	0.000
	(0.003)	(0.003)	(0.004)
retweet	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
reply	0.000	−0.000	−0.000
	(0.000)	(0.000)	(0.000)
likes	0.000	0.000	−0.000
	(0.000)	(0.000)	(0.000)
Constant	1.352^{***}	1.358^{***}	1.075^{***}
	(0.219)	(0.216)	(0.236)
Observations	2,342	2,342	2,342
R^2	0.341	0.342	0.349
Adjusted R^2	0.339	0.339	0.346
Residual Std. Error	0.776 (df = 2332)	0.776 (df = 2331)	0.772 (df = 2330)
F Statistic	134.347^{***} (df = 9; 2332)	121.201^{***} (df = 10; 2331)	113.790^{***} (df = 11; 2330)
AIC	5,472.153	5,471.900	5,447.470
BIC	5,535.499	5,541.006	5,522.334
P-value from robustified portmanteau	0.003	0.553	0.003
Note:			*p<0.1; **p<0.05; ***p<0.01

Lastly, the three final models combine the variables: the benchmark models + tweets, retweets, replies, and likes + Google search volumes for both Elon Musk and Tesla. From what has been presented, these models can be considered complete, with all variables relevant to the research question and the goal of this thesis.

As expected, the first three "benchmark-variables" are significant, and none-notable changes, similar to the other models presented. In model 12, the Elon Musk Google trend returns significance with a p-value of less than 0.01, but not the Google trend variable for Tesla, which also returned significance in model 6 (Table 6). When every variable is included, it could seem that interest towards Elon Musk calms the market (decreases volatility).

Adjusted R^2 has increased, which means these models explain the dependent variable better than the benchmark models do. Model (10) and (11) also has a decrease in AIC and BIC, which again tells us the models are better than the benchmark models. Model (12), on the other hand, has a slight increase in AIC but a decrease in BIC. As the AIC is only up from 5 446.528 to 5 447.470 we consider it similar enough.

4.4 Summary results

We have presented 12 different models, and now we summarize the key insights from the various specifications.

The presented models are similar in terms of R-squared, adjusted R-squared, and the AIC and BIC estimations. Simply put, the various goodness-of-fit measures are similar, but there are differences, and the models themselves are different. First, the benchmark models, only include the range-based volatility estimators and the leverage and VIX variables. Second, we introduce the Google Trend variables for the search terms "Elon Musk" and "Tesla" denoted as $GTEM$ and GTT. Thirdly, the Twitter data was added, with four variables: *tweet*, *retweet*, *reply*, and *likes*. Lastly, we combined all of the aforementioned variables with the benchmark models and presented the complete models, where all the variables used in this study were included. In the following discussion, the variables and models included will be further elaborated on in the next chapter.

5 Discussion

Most of the crises mentioned in the introduction have resulted in consequences that reached far longer than the market and country of origin. They play a large part in everyday life, even if you are not interested in trading stocks or commodities. Whether it is like the dot-com bubble, where speculative trading in technology stocks is the starting point, or whether something else in the economy causes the crisis, most often it impacts the global economy. With such a large impact on each other, it is crucial to determine if one person can either manipulate or affect the market in any direction.

The objective of this thesis is to examine whether Elon Musk's public statements on Twitter affect the U.S. equity market. The concrete research we have set out to answer reads:

Is there an Elon Musk effect on the U.S. Equity market?

Where the Elon Musk effect refers to when the interest in Elon Musk can be associated with future price movements. From our initial data exploration, we learned that some of our variables had a long memory, meaning that the time series itself drives new values.

Elon Musk's Twitter activity got good responses in the form of likes, retweets, and replies, which are correlated with his tweeting again. This makes sense, considering that if he didn't get any response, why post more? And if he got a response, he would engage with his followers more as he had gotten some response. It has not been tested, but it is natural to think that more activity from Elon Musk's Twitter also results in more followers, responses, likes, and retweets. Opposite as well, where less activity could result in losing followers from the account.

The Google search volumes also had considerable auto-correlation in lags 1, 5, and 22. This lends credibility to the idea that there is only this and that much information a person can consume in line with the Limited Attention theory. So, what drives people to search for either "Elon Musk" or "Tesla" is observed and executed at different times. The expectation was that people would react to Elon Musk's tweets by searching for relevant information (using Google) and that it could be expected that significant variables could be seen, confirming the suspicion. The correlation between "tweet" and the Google Trend search term "Elon Musk" is 0.332, and between "tweet" and the search term "Tesla" returned 0.386. Further, it confirmed the expectation that there would be a relationship between the two. Even though the correlation and relationship were not as high as thought out at first. From the results section, one can see that the models with Google trend variables for the search term "Tesla" included, returns significant on a significance level of 10%, which is OK. The variables were insignificant for the search term "Elon Musk" compared to the same models (4,5, and 6).

It was introduced 12 models, where the three first models, HAR (1), HAR + Leverage (2), and HAR + Leverage + VIX (3) are the benchmark models for this study. These models have variables that have a significant relationship to the dependent variable within a significance level of 1%. The variables are the average of the volatility estimators considered for the Tesla stock. As the variables are significant, they have an impact on the dependent variable (RB_t), which in this case is the volatility estimator. This is the average between two estimators, Garman and Klass, and Rogers and Satchell.

The next three models, 4,5, and 6, introduced the Google Trend variables for the search terms "Elon Musk" and "Tesla." Within these models, they all return significant values for "Tesla", on a 10% significance level. Model 6 also returned "Elon Musk" as significant, with a p-value of 0.05. An interesting finding is that the search term "Tesla" has a significant relationship with the dependent variable, which is the volatility estimator. Simply put, if people search for Tesla on Google, the volatility estimator is affected by it. 2‰ is not a big impact, but it still confirms the suspicion that there is a significant relationship between the two.

Models 7, 8, and 9 are similar, but with Twitter Variables. Here, there were no significant relationships between the variables and the volatility estimator. In other words, Elon Musk's tweets have no direct relationship with volatility. This could be because the data sample of tweets we used, includes a lot of tweets that either are not about Tesla or do not contain any important information that could result in volatility changes in the stock price.

The final models presented were models where both Google trend- and Twitter variables were included. However, only one of the six variables had a significant effect on the volatility estimator (in model 12), and the search term "Elon Musk" showed significance with a p-value of 0.01. With everything considered (Google search volumes and tweets), there is a relationship between Google searches for Elon Musk and the volatility estimator. Meaning, that if there is one unit increase for the search term "Elon Musk", the volatility estimator will decrease by 0.001%. That is a negative relationship, increased interest in Elon Musk decreases the volatility in the Tesla stock price.

Considering the models, the ones with one or more significant relationships with the dependent variable also return good values for AIC, BIC, and p-value from the robustified Portmanteau test. As the models are considered OK, based on the tests. We can confirm that there is a relationship between Google searches for either Tesla or Elon Musk and volatility for the Tesla stock. At least according to the data and models that are processed during this study. this can be seen in table 6, where there is a significant relationship between both the search terms and the dependent variable. Nadeem (2021) and the article, one argument for the results is that the examples and similar incidents do not occur often enough to generate a significant impact on the volatility. The data set is for a longer period of 10 years, and the examples in the article from Nadeem (2021) range from 2018 to 2021. In that time period, there were 16 examples where there was some impact, either on the Tesla or on others. Nadeem (2021) had three occurrences with an impact on the Tesla stock price. Simply put, within the time frame of three years, there were three times when his tweet affected the stock. An argument against people buying stocks based on Elon Musk's tweets is that people might consider his tweets as not serious or jokes, as Elon Musk considers himself a "Troll" on the internet, which is a nickname for joker, comedian, etc. He regularly posts jokes and other un-serious content that is not relevant to his position at Tesla, nor has anything to do with Tesla.

Following the finding by Han and Yang (2013), we assume that the results from this study correspond with their conclusion. Several other variables impact the stock price volatility, such as news, financial reports, and similar. This might be one of the reasons why the results did not return significant relationships, or higher impact ones, between the independent and dependent variables.

Another aspect is that Elon Musk, back in 2011, did not have as many followers as he has today, and further would struggle to gain the same attention towards his tweets as he can today.

The analyses, models, and results are based on the assumption that Elon Musk's Twitter activity causes Google search volume spikes, which was tested and confirmed in Table 4. Numerous other factors could cause attention towards Tesla or Elon Musk. Tesla, as a company, has announcements and news that might affect Google's search volume, especially for the term "Tesla". Tesla, as a car manufacturer, also attracts the attention of possible buyers, who probably would like to read up on different models and specifications before buying.

Other insights gained through the data

This thesis is partly data-driven and some interesting insights that are not directly linked to the research question have emerged during the process.

The Google search volume for "Elon Musk" influences Elon Musk's Twitter use.

The Google search volume for "Tesla" influences Elon Musk's Twitter use.

Many of the key variables have a considerable lag, leading to the assumption that there is a correlation between different sets of lagged variables of the attention markers and the volatility of the stock.

As mentioned in chapter 2.3.5, this study aims to contribute to existing literature in three ways. That the attention between the tweets of Elon Musk and Google search volumes are related, that the attention to Tesla is related to Tesla's price variation, but not attention to Elon Musk, and lastly, that Google search volumes, in general, are more useful as one of the leading indicators of Tesla's price variation.

That can be seen from the models (1-12) under results, where the relationships mentioned have significant returns. This confirms that there is a relationship between tweets and Google search volumes, Tesla and the stock price variation, and Google search volume is generally more useful as the leading indicator for Tesla's price variation.

Discussion Ethical and legal aspects of the research

As stated in Chapter 3 earlier, as a researcher, one must adhere to various laws, regulations, and ethical standards when conducting research. For this work done with this thesis, the GDPR and research ethics were considered and worked with as part of the initial work. As we did not collect any new data or personal information, we did not need permission from SIKT. This is a requirement if one is to conduct research that contains personal information at Inland Norway University of Applied Science or in Norway in general.

The research was based on publicly available tweets; thus, we can use them without further permission. The person posting them must have known they could be used, reused, etc. The ethical part concluded that even though this research is based on a person, no harm should be derived from it. Thus, it was okay to conduct this research both ethically and legally.

Discussion around the different hypothesis

There was a single research question: "Is there an Elon Musk effect on the U.S equity market?". From that, three different hypotheses were derived. Those where:

H_1: Elon Musk's Twitter use influences the Google search volume for "Elon Musk".

From the correlation matrices presented in Table 2, it is clear that the tweets are correlated with the Google search volumes for "Elon Musk" with a factor of 0.57 (spearman) and 0.33 (Pearson). This along with the results in Table 3, model 1 where we test how the OLS regression with the Google search volume for "Elon Musk" was used as the dependent variable, and the lagged various Twitter activities and the Google search volume for "Elon Musk" used as independent variables showed that there was indeed a significant correlation between the lagged attention measures and the Google search volume for "Elon Musk".

H_2: Elon Musk's Twitter use influences the Google search volume for "Tesla".

From the correlation matrices presented in Table 2, it is clear that the tweets are correlated with the Google search volumes for "Tesla" with a factor of 0.56 (spearman) and 0.38 (Pearson). This along with the results in Table 3, model 1 where we test how the OLS regression with the Google search volume for "Tesla" was used as the dependent variable, and the lagged various Twitter activities and the Google search volume for "Tesla" used as independent variables showed that there was indeed a significant correlation between the lagged attention measures and the google search volume for "Tesla".

H_3: The volatility of Tesla's stock price is influenced by Elon Musk's Twitter use.

From Tables 7 and 8. We can see that his Twitter use does not impact the volatility of the stock directly. However, from Table 8, model 12, it is observable that even with robust coefficients, the Google search volume for “Elon Musk” is significant. And his Twitter use is, as seen in Table 3, significant for the Google search volumes for “Elon Musk”.

H_4: The volatility of Tesla’s stock price is led by Google search volume for “Elon Musk” or “Tesla”.

From table 6, we can see that there is a significant relationship between the Google Trend variables and the volatility estimator. Whereas, one can conclude that the stock price volatility of Tesla is (at least) partially driven by Google search volume for “Elon Musk” and “Tesla”.

Bias

We acknowledge that we, as researchers, both experience and contribute to biases in our research. These can influence how we collect data, how we understand the results, and whether we started our research in the right place. There could be that the data we used was wrong, or misleading, and that we did not understand the data well enough.

The data collected was from Kaggle, google search volume, and Yahoo finances. The biggest downside of this was the data collected from Kaggle, as we intended to use data directly from Twitter, to have the most updated data. However, since this has been placed behind a paywall. We didn’t have the means to get the data directly anymore. By doing it this way, rather than purchasing the data directly from Twitter, we also make the study reproducible by us or others. There are many other ways to study this and locate other findings and interpretations of the data. Similarly, by doing it this way, and others want to research the same, the research might improve as a result.

All the data is publicly available and should be available for further research and to validate the results. However, we acknowledge that the lack of the latest data could slightly offset the results of this thesis. However, considering there was data for close to 10 years, we feel confident in the results.

5.1 Suggestions for further research and limitations

Extending our study could be done in many ways, first one could include testing in terms of more or other combinations of Google search terms. As one is searching for something on Google, it is rarely high-pressure, and errors can occur. When searching for names like "Elon Musk", some prefer to write the last name first, or only the last name, etc. There are, for example, more searches for only "Musk" than for "Elon Musk". Therefore, one direction for future research is to experiment with more phrases or combinations of phrases to include every search for what one is looking for. The dataset contains tweets, and other variables from 3.11.2011 until 16.04.2021. In that time frame, a lot happened to the number of followers he had on Twitter. Another direction could be to test smaller samples, preferably later years, where he has more followers, e.g., 2018 to 2021, where the examples from Nadeem (2021) are from. Or, the other way around, that the sample period could be longer. A limitation of our study was that Twitter implemented a payment wall for their API. Which, as mentioned, led us to use only the data downloaded from Kaggle. One could also consider to include other social media or discussion forums. This way, one could quantify attention in a more concrete way than what has been done in our study.

Another possibility is only to investigate the relationship between the tweets and google search volumes (search trends) to determine how much attention Elon Musk can gather from his tweets. Similarly, one could look into the volatility of the Tesla stock based on Google search volumes. This research is based on the assumption that the tweets cause Google search volume spikes, but the Google search volumes could also be a result of other things, such as launches of new models or other announcements directly from Tesla.

Another interesting question is how markets would react to other public figures, e.g., Donald Trump or Mark Zuckerberg. In recent years, both as president of the U.S. and not, Donald Trump has gathered a lot of media attention and controversy regarding his Twitter usage, especially.

This thesis's basis is from former research within social media, equity-market, volatility measures, and similar, general financial terms and theories that have been connected to the literature as best possible. Due to our study's time limitation, we could not consider every part of this research area.

6 Conclusion

The motivation for this thesis was the article by Nadeem (2021), where we wanted to find out if there are more times, and enough times, that Elon Musk has impacted the U.S. stock market. If it was possible to determine a significant relationship between Elon Musk's tweets and the volatility of the Tesla stock price. Throughout this study, many interesting findings have been uncovered.

As for how we contribute to the existing literature, we examine how Elon Musk's Twitter activity ties in with Google search volumes towards "Elon Musk" and "Tesla." And how these search terms and the Google search volumes correlate with the Tesla stock's volatility. As far as we can tell, this has not been done before.

Relationships between Google search volume and volatility, as well as between tweets and Google trends, have been found. The range-based estimators calculate the volatility regarding day-to-day changes. During the data exploration portion of the research, significant autocorrelation lags were discovered in the Google search volume for "Tesla."

The key findings were that Elon Musk's use of Twitter led the Google search volumes of both keywords "Elon Musk" and "Tesla" and that his use of Twitter was statistically significant towards the volatility estimators. Relationships between tweets and Google search volume and Google search volume and volatility have been discovered.

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